Winter '22 Exam 2

1) a) addition and scaler multiplication

b) polynomials with real coefficients

c) The determinant is zero

2) a) $\begin{bmatrix} \cos(-4\pi/3) & -\sin(-4\pi/3) & 0 \\ \sin(-4\pi/3) & \cos(-4\pi/3) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 9 \\ 0 & 1 & 12 \\ 0 & 0 & 1 \end{bmatrix}$

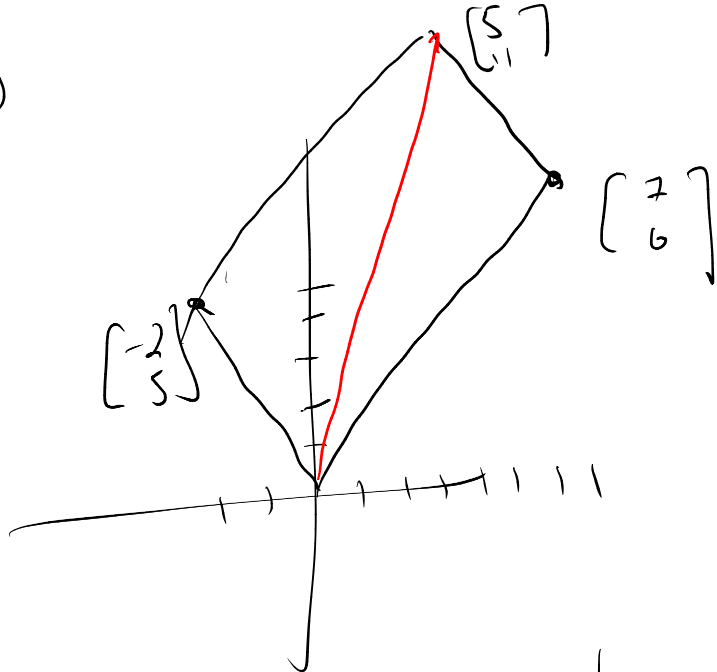
c) $\begin{bmatrix} 1/4 & 0 & 6 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

d) $C \cdot B \cdot A$

3) a) $\det(A) = -12 - 35 = -47 \neq 0$

so A is invertible

b)



$$\text{Area} = \left| \det \begin{bmatrix} -2 & 7 \\ 5 & 6 \end{bmatrix} \right|$$

$$= \left| \det \begin{bmatrix} -2 & 5 \\ 7 & 6 \end{bmatrix} \right| = 47$$

4)

a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\quad a = -b + 3d - 4c$

$\begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 \\ 3 & 4 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad a + 4c = 1$

$-b + 3d = 3$ not equal

c) From a), we know W is nonempty.

write $\begin{bmatrix} a & b \\ c & d \end{bmatrix}, \begin{bmatrix} x & y \\ z & w \end{bmatrix} \in W$

then $a+4c = -b+3d$,

$x+4z = -y+3w$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} x & y \\ z & w \end{bmatrix}$$

$$= \begin{bmatrix} a+x & b+y \\ c+z & d+y \end{bmatrix}$$

$$(a+x) + 4(c+z)$$

$$= a + 4c + x + 4z$$

$$= -b+3d - y + 3w$$

$$= -(b+y) + 3(d+w)$$

Now if $k \in \mathbb{R}$,

$$k \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

$$ka + 4kc = k(a+4c)$$

$$= k(-b+3d)$$

$$= -(kb) + 3\,kd$$

So W is a subspace of $M_2(\mathbb{R})$.

5)

Note $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \in S$ since

$0 \cdot 0 - 0 = 0$.

Now take $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \in S$.

$$3 \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

$3 \cdot 3 - 3 = 6 \neq 0$

so $3 \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \notin S$

and so S is not a subspace of $\mathbb{R}^3$

Note $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \in S$ since

$0 \cdot 0 - 0 = 0$.

The only possible subspaces of $\mathbb{R}^3$ are points, lines, planes, + $\mathbb{R}^3$.

Now take $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \in S$. ($S$ is not a point)

Also $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \in S$, so S is not a line.

Since $\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} \notin S$, S is not $\mathbb{R}^3$

But also $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \notin S$

since $2 \cdot 1 - 1 = 1 \neq 0$

So S is not a plane.