Exam 3 Winter '22

a) $0 = \det(A - \lambda I_2)$

$= \det\left(\begin{bmatrix} -4 & 2 \\ -4 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)$

$= \det\left(\begin{bmatrix} -4-\lambda & 2 \\ -4 & 5-\lambda \end{bmatrix}\right)$

$= (-4-\lambda)(5-\lambda) + 8$

$= \lambda^2 - \lambda - 12$

$= (\lambda - 4)(\lambda + 3)$

$\lambda = 4, -3$

b)

$$\begin{bmatrix} -4 & 2 \\ -4 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -3 \end{bmatrix} = -3 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 2 \\ -4 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 16 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

2) a) $y = mx + b$

$7 = b$

$3 = 6m + b$

$-2 = 5m + b$

$1 = 3m + b$

b) $\begin{bmatrix} 0 & 1 \\ 6 & 1 \\ 5 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ -2 \\ 1 \end{bmatrix}$

c) $A^tA = \begin{bmatrix} 0 & 6 & 5 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 6 & 1 \\ 5 & 1 \\ 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 70 & 14 \\ 14 & 4 \end{bmatrix}$

$$A^t \vec{b} = \begin{bmatrix} 0 & 6 & 5 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ -2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 11 \\ 9 \end{bmatrix}$$

d)

$$\begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 70 & 14 \\ 14 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 11 \\ 9 \end{bmatrix}$$

$$= \frac{1}{84} \begin{bmatrix} 4 & -14 \\ -14 & 70 \end{bmatrix} \begin{bmatrix} 11 \\ 9 \end{bmatrix}$$

$$= \frac{1}{84} \begin{bmatrix} -82 \\ 476 \end{bmatrix} = \begin{bmatrix} -\frac{41}{42} \\ \frac{119}{21} \end{bmatrix}$$

$$y = -\frac{41}{42} x + \frac{119}{21}$$

3) a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = A$

b) $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} = A'$

$$B = \begin{bmatrix} 0 & 1/3 & 1/2 \\ 0 & 1/3 & 1/2 \\ 1 & 1/3 & 0 \end{bmatrix}$$

c) $C = \frac{17}{20} B + \frac{3/20}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$$C = \begin{bmatrix} 1/20 & 1/3 & 19/40 \\ 1/20 & 1/3 & 19/40 \\ 9/10 & 1/3 & 1/20 \end{bmatrix}$$

d) $\lambda = 1$

e)

$$\frac{57}{57+57+74} = \frac{57}{188} \approx .30319$$

4) a) $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}2\\0\\1\end{bmatrix}$

$$T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}-1\\1\\0\end{bmatrix}$$

$$A = \begin{bmatrix}2 & -1\\0 & 1\\1 & 0\end{bmatrix}$$

b) $\mathrm{Ran}(T) = \mathrm{Col}(A) = \mathrm{span}\left\{\begin{bmatrix}2\\0\\1\end{bmatrix}, \begin{bmatrix}-1\\1\\0\end{bmatrix}\right\}$

since $\begin{bmatrix}2\\0\\1\end{bmatrix}$ & $\begin{bmatrix}-1\\1\\0\end{bmatrix}$ are

not multiples of eachother,

$\mathrm{Ran}(T)$ is a plane.

c) If $T\left(\begin{bmatrix}x\\y\end{bmatrix}\right) = \begin{bmatrix}0\\0\\0\end{bmatrix}$,

$$\begin{bmatrix}2x-y\\y\\x\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

So $x = y = 0$.

Then $\ker(T) = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$, so the orthogonal projection onto $\ker(T)$ is the zero matrix $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.