Math 413/513 Assignment 1

Due Tuesday, September 16

1) Let $n \in \mathbb{N}$ and let $\{x_i\}_{i=1}^n$ and $\{y_i\}_{i=1}^n$ be positive real numbers. Use induction to show

$$\sum_{i=1}^n \sqrt{x_i y_i} \leq \left(\sum_{i=1}^n x_i\right)^{1/2} \left(\sum_{i=1}^n y_i\right)^{1/2}.$$

2) Let S, T, and U be sets and let $f : T \to U$, $g : S \to T$.

a) Prove that if $f \circ g$ is bijective, then f is surjective and g is injective.

b) If f is injective and g is surjective, does it then follow that $f \circ g$ must be bijective? Prove or give a counterexample.

c) Verify that if S is finite, then any injection from S to itself is also surjective.

d) Give an example of an injection $f : \mathbb{R} \to \mathbb{R}$ that is not surjective.

3) (Remark 2, page 2) Let V be a vector space over $\mathbb{R}$ (or, if you like, $\mathbb{C}$).

a) Show that 0_V is unique.

b) If $v \in V$, show that $-v$ is unique.

c) Prove that, for all $v \in V$, $0_V = 0 \cdot v$.

d) Finally, establish that for all $v \in V$, $-v = (-1) \cdot v$.

4) This problem will test both your ability to construct a reasonable argument and your compliance with definitions: prove that $\mathbb{C}$ is a vector space over $\mathbb{R}$.

5) (Problem 1.2, c)) Let $n \in \mathbb{N}$. Prove that the set of all polynomials of degree exactly n with real (or complex, if you like) coefficients is NOT a vector space over $\mathbb{R}$ (or $\mathbb{C}$).

$$D(\mathbf{v}_1, \ldots, \underbrace{\mathbf{v}_k + \mathbf{u}}_{k}, \underset{k}{\mathbf{v}_k}, \ldots, \mathbf{v}_n)$$