1) Fix $A \in M_n(\mathbb{R})$

$$A\, 0_{M_n(\mathbb{R})} = 0_{M_n(\mathbb{R})} = 0\, A_{M_n(\mathbb{R})}$$

so $0_{M_n(\mathbb{R})} \in W_A$

Now let $B, C \in W_A$, $\alpha \in \mathbb{R}$

Then $AB = BA$, $AC = CA$, so

$$\begin{aligned}
&A(\alpha B + C) \\
&= A(\alpha B) + AC \\
&= \alpha AB + AC \\
&= \alpha BA + CA \\
&= (\alpha B)A + CA \\
&= (\alpha B + C)(A)
\end{aligned}$$

Then $\alpha B + C \in W_A$

$\Rightarrow$ W_A is a subspace by the subspace test

2) Let $g, f, h \in \mathcal{F}(\mathbb{R})$, $\alpha \in \mathbb{R}$

Then

$$T_f(\alpha g + h)(x)$$

$$= ((\alpha g + h) \circ f)(x)$$

$$= (\alpha g + h)(f(x))$$

$$= (\alpha g)(f(x)) + h(f(x))$$

$$= \alpha(g(f(x)) + h(f(x))$$

$$= \alpha(g \circ f)(x) + (h \circ f)(x)$$

$$= \alpha T_f(g)(x) + T_f(h)(x)$$

and so T_f is linear

3) i) Suppose T is injective and $\mathcal{B}$ is linearly independent.

Let $x_1, -, x_n \in \mathcal{B}$ and suppose $\exists\ \alpha_1, -, \alpha_n \in \mathbb{R}$

$$\alpha_1 T(x_1) + \alpha_2 T(x_2) + - + \alpha_n T(x_n)$$
$$= \sum_{i=1}^{n} \alpha_i T(x_i) = 0_W$$

Then by linearity of T,

$$0_W = \sum_{i=1}^{n} \alpha_i T(x_i) = \sum_{i=1}^{n} T(\alpha x_i)$$
$$= T\left(\sum_{i=1}^{n} \alpha x_i\right)$$

By injectivity of T,

$$\sum_{i=1}^{n} \alpha_i x_i = 0_v$$

But since $\mathcal{B}$ is linearly independent,

$$\alpha_1 = \alpha_2 = \dots = \alpha_n = 0$$

and so $T(\mathcal{B})$ is linearly independent

(ii) Let $\{v_1, v_2, \ldots, v_n\}$ be a basis.

Then if $x \in V$, $\exists\ \alpha_1, \ldots, \alpha_n \in \mathbb{F}$

$$x = \sum_{i=1}^{n} \alpha_i v_i$$

Now let $\beta_i = \frac{\alpha_i}{i}$

Then $\sum_{i=1}^{n} \beta_i i v_i$

$$= \sum_{i=1}^{n} \frac{\alpha_i}{i} i \cdot v_i$$

$$= \sum_{i=1}^{n} \alpha_i v_i = x$$

Now suppose

$$x = \sum_{i=1}^{n} \gamma_i (i v_i)$$

for some $\gamma_1, \gamma_2, \ldots, \gamma_n \in \mathbb{F}$

Then $x = \sum_{i=1}^{n} (\gamma_i \cdot i) v_i$

$\Rightarrow$ by uniqueness,

$$\gamma_i \cdot i = \alpha i$$

and so,

$$\gamma_i = \frac{\alpha i}{i} = \beta i$$

Therefore $\{v_1, 2v_1, \ldots, nv_n\}$ is a basis.