So $\dim_{\mathbb{R}}(\mathbb{R}^2 \otimes \mathbb{R}^2) = 4$

$\Rightarrow$ $\mathbb{R}^2 \otimes \mathbb{R}^2$ is isomorphic to $\mathbb{R}^4$

as a vector space.

$\dim_{\mathbb{R}}(\mathbb{R}^2 \otimes \mathbb{R}^2 \otimes \mathbb{R}^2) = 8$, and in general,

$$\dim_{\mathbb{R}}(\underbrace{\mathbb{R}^2 \otimes \mathbb{R}^2 \otimes \cdots \otimes \mathbb{R}^2}_{n \text{ times}}) = 2^n$$

# Dimension of a Tensor Product

Suppose $V_1, V_2, \ldots, V_n$ are finite dimensional vector spaces over $\mathbb{R}$. Then

$$\boxed{\dim_{\mathbb{R}}(V_1 \otimes V_2 \otimes \cdots \otimes V_n) = \dim_{\mathbb{R}}(V_1) \cdot \dim_{\mathbb{R}}(V_2) \cdots \dim_{\mathbb{R}}(V_n)}$$

## Definition: (Simple tensors)

Let $V_1, V_2, \ldots, V_n$ be vector spaces over $\mathbb{R}$ and let

$$f_i \in (V_i^*)^* \quad \forall\ 1 \le i \le n$$

Define $f_1 \otimes f_2 \otimes \cdots \otimes f_n$

$\in V_1 \otimes V_2 \otimes \cdots \otimes V_n$ by

$$\boxed{\begin{aligned}&(f_1 \otimes f_2 \otimes \cdots \otimes f_n)((x_1, x_2, \ldots, x_n)) \\ &= f_1(x_1) f_2(x_2) \cdots f_n(x_n)\end{aligned}}$$

Any such element in $V_1 \otimes V_2 \otimes \cdots \otimes V_n$

is called a simple tensor

Warning: not every element in

$V_1 \otimes V_2 \otimes \cdots \otimes V_n$ is

a simple tensor! (HW)