Math 473/573 Assignment 1

Due Thursday, January 23

1) For each matrix, use your favorite program (or your hands, if you are masochistic) to approximate the eigenvalues to 4 decimal places. Then determine whether the matrix is invertible and find an approximate formula for the inverse, if it exists.

a) $A = \begin{bmatrix} 3 & -5 \\ 2 & 1 \end{bmatrix}$

b) $B = \begin{bmatrix} 0 & 1 & 4 \\ -8 & 6 & -2 \\ 7 & 0 & 2 \end{bmatrix}$

c) $C = \begin{bmatrix} -1 & 1 & 3 & 0 \\ 9 & 6 & 4 & 0 \\ -11 & 0 & 5 & 2 \\ 0 & -3 & 5 & 13 \end{bmatrix}$

2) Suppose you want to find a quadratic $y = ax^2 + bx + c$ that interpolates

$$(1, 2), \ (2, 0), \ \text{and} \ (3, 1).$$

a) By plugging each point into $y = ax^2 + bx + c$, write out a system of three equations in three unknowns, in matrix form, that could be used to find a, b, and c.

b) The command "fliplr(vander(1:n))" in MatLab gives the $n \times n$ Vandermonde matrix associated to the values $x_1 = 1, x_2 = 2, \ldots, x_n = n$. Either use this or flat-out input the matrix to solve the system in part a) for a, b, and c. The Matlab command for solving $Ax = b$ is $A\backslash b$ where x and b are *column* vectors- so be careful!

c) Graph the points and your quadratic on the same axis.

d) From class, we know that an $n \times n$ Vandermonde matrix is always invertible provided none of the $x_i's$ are repeated. Calculate the eigenvalues of a 10×10 Vandermonde matrix A, corresponding to $x_i = i$ for $1 \leq i \leq n$, to four decimal places. What does this appear to tell you about the invertibility of A?

e) Carry out the same experiment with 16 decimals places. What does this say about invertibility of A now?

3) Problem 1.1 in the text.

4) Problem 1.4 in the text.

5) Let A be a 137×5 complex matrix and suppose the columns of A, written as $a_1, a_2, a_3, a_4, a_5 \in \mathbb{C}^{137}$ satisfy

$$2a_1 - ia_2 + 15a_3 + (2 + 5i)a_4 - 1001a_5 = \vec{0}$$

where $\vec{0}$ is the zero vector in $\mathbb{C}^{137}$. Find a nonzero vector in $\ker(A)$.