Math 473/573 Assignment 2

Due Tuesday, February 4

1) Let

$$v = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \; w = \begin{bmatrix} i \\ -1+i \\ 1 \end{bmatrix}$$

a) Calculate the one-, two-, and infinity-norms of both vectors exactly, using any program at your disposal.

b) Find a unit vector (in 2-norm) orthogonal to v.

c) Find a unit vector (in 2-norm) orthogonal to both v and w.

2) For each matrix, calculate the one- and Frobenius norms exactly, then determine the reduced and full singular value decomposition of each matrix up to four decimal places and use it to calculate the 2-norm of the matrix, again up to four decimal places. For part a), change one coordinate so that the resulting matrix's eigenvalues are the same (in modulus) as its singular values.

a) $A = \begin{bmatrix} 3 & -5 \\ 2 & 1 \end{bmatrix}$

b) $B = \begin{bmatrix} 0 & 1 \\ -8 & 6 \\ 7 & 0 \end{bmatrix}$

3) Problem 3.2 in the text.

4) Problem 4.2 in the text.

5) Problem 4.4 in the text.

6) Choose your favorite (nonzero) number z and let v be the vector in $\mathbb{C}^2$ whose first coordinate is z and whose second is z^2.

a) Construct a unitary matrix Q whose first column is a vector in $span(v)$ and whose second column is a vector in $span(v)^{\perp}$.

b) Let $S = \begin{bmatrix} z & 4 \\ z^2 & 5 \end{bmatrix}$. Calculate the image under S, in 2-norm, of the 2-norm unit ball in $\mathbb{C}^2$.

7) If $A \in \mathbb{C}^{n \times n}$ and A is invertible, show that $|A|$ is invertible.