Math 473/573 Assignment 3

Due Tuesday, February 18

1) For each matrix, calculate the reduced and full QR decomposition up to four decimal places.

a) $A = \begin{bmatrix} i & -3 \\ 2+i & 16 \end{bmatrix}$

b) $B = \begin{bmatrix} 5-i & \sqrt{2} \\ -11 & 4 \\ 8i & 32 \end{bmatrix}$

2) Problem 6.1 in the text.

3) Problem 7.4 in the text.

4) Problem 8.1 in the text.

5) Problem 9.1 in the text.

6) Recall that one-dimensional subspaces of $\mathbb{R}^2$ are just lines through the origin.

a) For every such line ℓ, find a matrix P in the standard basis for the orthogonal projection onto ℓ. Your answer should depend on the slope of ℓ.

b) Now consider the basis $\{v_1, v_2\}$ where v_1 is a unit vector on the line ℓ and v_2 is a unit vector on the line perpendicular to ℓ. Find the matrix of the orthogonal projection onto ℓ in the basis $\{v_1, v_2\}$.

c) Now choose your favorite line ℓ through the origin that is neither vertical, horizontal, nor $y = x$. In the standard basis, find the matrix of one NON-orthogonal projection onto ℓ.

7) Let

$$x = \begin{bmatrix} 1 \\ \sqrt{2} \end{bmatrix}, y = \begin{bmatrix} 0 \\ \sqrt{3} \end{bmatrix}.$$

a) Set $v = x - y$ and let $F = I_2 - \frac{2}{v^*v}(vv^*)$. Show that $Fx = y$ and $Fy = x$, by Matlab or any other computational resource, if you like.

b) Pick another vector z with $\|z\|_2 = \|x\|_2$ and set $v = x - z$. Show that $Fx = z$ and $Fz = x$, again using a computational resource, if you like.

c) Explain why whenever $\|x\|_2 = \|y\|_2$ and $v = x - y$, then we must have $Fx = y$ and $Fy = x$. *Hint:* Draw a picture.

8) If $A \in \mathbb{C}^{m \times n}$ and $A = QR$ is the full QR decomposition of A, show that $\|A\|_2 = \|R\|_2$.