Math 473/573 Assignment 4

Due Thursday, March 27

1) Problem 13.4 in the text. Some hints:

a) Remember that Newton's method works as follows: With initial guess x_0,

$$x_1 = x_0 - \frac{p(x_0)}{p'(x_0)}, \quad x_2 = x_1 - \frac{p(x_1)}{p'(x_1)}, \quad x_3 = x_2 - \frac{p(x_2)}{p'(x_2)}, \text{ etc.}$$

b) If you've never used Mathematica before, you can access it on any computer on the 2nd floor of CASL. Here are some inputs that might help:

Defining p: p[x_] = x^5 - 2*x^4 - 3*x^3 + 3*x^2 - 2*x - 1

Defining the derivative of p: q[x_] = D[p[x], x]

Setting up Newton's method: f[x_] = x - (p[x]/q[x])

Then f[0] gives x_1, $f[x_1]$ gives x_2, etc. Be careful not to use underscores in Mathematica for the x_i's, use $x1, x2$,etc.

2) Problem 14.2 in the text. Some hints:

a) Multiply out $f(\varepsilon_{machine}) = (1 + O(\varepsilon_{machine})(1 + O(\varepsilon_{machine})$. The first $O(\varepsilon_{machine})$ is less than or equal to $k_1\varepsilon_{machine}$ and the second $O(\varepsilon_{machine})$ is less than or equal to $k_2\varepsilon_{machine}$ for some $k_1, k_2 > 0$. Then remember that as $\varepsilon_{machine}$ goes to zero, $\varepsilon^2_{machine} \leq \varepsilon_{machine}$.

b) Cross-multiply.

3) Problem 15.2 in the text: just do a) and c).

4) Define the third and fourth Hilbert matrices by

$$H_3 = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix} \;\&\; H_4 = \begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 \\ 1/2 & 1/3 & 1/4 & 1/5 \\ 1/3 & 1/4 & 1/5 & 1/6 \\ 1/4 & 1/5 & 1/6 & 1/7 \end{bmatrix}$$

a) Find the condition numbers $\kappa(H_3)$ and $\kappa(H_4)$ to four decimal places with respect to the 2-norm.

b) If $b = [1, 1, 1]^t$ and $b' = [1, 1, 1.01]^t$, solve the systems

$$H_3 x = b, \quad H_3 x' = b'$$

using your favorite program. Then find the absolute and relative errors with respect to the 2-norm.

c) If $b = [1, 1, 1, 1]^t$ and $b' = [1, 1, 1, 1.01]^t$, solve the systems

$$H_4 x = b, \quad H_4 x' = b'$$

using your favorite program. Then find the absolute and relative errors with respect to the 2-norm.

5) This problem shows rigorously that floating-point multiplication is backwards stable. Let $x, y \in \mathbb{C}$.

Recall that there are floating point approximations $fl(x) = (1+\varepsilon_1)x$ and $fl(y) = (1+\varepsilon_2)y$ for y for some $\varepsilon_1,$, ε_2 with $|\varepsilon_1|,$, $|\varepsilon_2| < \varepsilon_{machine}$.

Further recall that $fl(x) \otimes fl(y) = (1 + \varepsilon_3)(fl(x) \times fl(y))$ for some ε_3 with $|\varepsilon_3| < \varepsilon_{machine}$

a) If $\tilde{f} : \mathbb{R} \times \mathbb{R} : \mathbb{R}$ is defined by $\tilde{f}((x, y)) = fl(x) \otimes fl(y)$, find $\tilde{x}, \tilde{y} \in \mathbb{R}$ with $\tilde{x} \times \tilde{y} = \tilde{f}((x, y))$.

b) Show that $\dfrac{|x - \tilde{x}|}{|x|} = O(\varepsilon_{machine})$ and $\dfrac{|y - \tilde{y}|}{|y|} = O(\varepsilon_{machine})$.

c) Finally, show that $\dfrac{\|(x, y) - (\tilde{x}, \tilde{y})\|_\infty}{\|(x, y)\|_\infty} = O(\varepsilon_{machine})$.