Math 473/573 Assignment 5

Due Tuesday, April 8

1) Problem 17.1 in the text, just do a) and c). Some hints:

What this problem wants if for you to take (17.4) and write out what the norms of R and δR are in terms of row and column sums. After you write it out, it wants you to apply (17.5) to get more descriptive upper bounds on the quotient $\|\delta R\|/\|R\|$.

For a), there has to be a specific column k for δR and a specific column ℓ for R where the norm is attained. Write that in. If $k = \ell$, then for any i, $1 \leq i \leq m$,

$$\frac{|\delta r_{i,k}|}{\sum_{i=1}^{m} |r_{i,k}} \leq \frac{|\delta r_{i,k}|}{r_{i,k}}, .$$

Apply this to every term of the sum in the numerator and then use (17.5). Now if $k \neq \ell$, then you can replace the column index ℓ in the denominator with the index k at the penalty of an inequality (be sure to tell me why you can do this!) and continue on as before.

For c), you can carry on as for a), but there is a trick that can get it done in one line. Think about how you relate the infinity norm of a matrix to the one norm.

2) Problem 18.1 in the text. Some hints:

a) and b), use Matlab with the short format. The answers you get will be exact.

e) Look at the box for theorem 18.1. For the top left, you'll want to pick an element δb in the range of A, so calculate the range by using the projection P you found in a). Then choose a nonzero vector (any vector, amazingly!) in the range of A to be δb and calculate

$$\frac{\|\delta y\|_2}{\|y\|_2} \cdot \frac{\|b\|_2}{\|\delta b\|_2},$$

which should give you the condition number exactly.

For the top right, you need δb to be a right singular vector for A^+, so compute the svd of A^+ using Matlab and choose δb to be the left-most column of V. Then compute

$$\frac{\|\delta x\|_2}{\|x\|_2} \cdot \frac{\|b\|_2}{\|\delta b\|_2}.$$

where $\delta x = A^+ \delta b$ and observe it is pretty close to the condition number- it may be exact in the short format.

For the lower right, compute the svd of A using Matlab. Let $p = \sigma_2 u_2$ where σ_2 is the smaller of the two singular values of A and u_2 is the second column of U in the svd of A. Let $\delta A = (\delta p) v_3^*$ where v_3 is the last column of V in the svd of A. Then compute

$$\frac{\|\delta y\|_2}{\|y\|_2} \cdot \frac{\|A\|_2}{\|\delta A\|_2}.$$

where $\delta y = A^+ b - (A + \delta A)^+ b$. Here you should only get a number that is fairly close to the condition number, don't worry about equality.

For the lower right, just put in a random vector and observe that the computed quantity

$$\frac{\|\delta x\|_2}{\|x\|_2} \cdot \frac{\|A\|_2}{\|\delta A\|_2}.$$

where $\delta x = AA^+ b - (A + \delta A)(A + \delta A)^+ b$ is less than the condition number. You get 5 points extra credit for accuracy up to 3 decimal places.

3) Calculate the exact number of flops used in algorithm 17.1.

4) Recall the third and fourth Hilbert matrices Define the third and fourth Hilbert matrices by

$$H_3 = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix} \; \& \; H_4 = \begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 \\ 1/2 & 1/3 & 1/4 & 1/5 \\ 1/3 & 1/4 & 1/5 & 1/6 \\ 1/4 & 1/5 & 1/6 & 1/7 \end{bmatrix}$$

a) write down H_5. If you don't see the pattern, let me know.

b) Write a program in Matlab that calculates both H_n for $n \geq 1$ and the condition numbers $\kappa(H_n)$. It should require no more than two "for" loops to define H_n.

c) Use your program to compute H_9 and $\kappa(H_9)$.