Math 473/573 Assignment 6

Due Thursday, April 17

1) Problem 20.4 in the text.

2) Problem 21.3 b) in the text. Some hints: in the factorization, U must be upper-triangular and L must be lower triangular with ones on the diagonal and $(2,1)$ entry of absolute value less than or equal to one. All you need is a 2×2 example where such a factorization won't work. There are precisely two permutation matrices in $M_2(\mathbb{C})$: the identity and $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. To choose your example, remember that a 2×2 matrix is singular if and only if one row is a multiple of the other.

3) Let $A = \begin{bmatrix} 1 & 1 & 2 \times 10^9 \\ 2 & -1 & 10^9 \\ 1 & 2 & 0 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, and $b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

a) Solve $Ax = b$ by hand using Gaussian elimination WITHOUT pivoting.

b) Solve $Ax = b$ by hand using Gaussian elimination WITH partial pivoting. Do you get a different answer than a)? Explain.

c) Use your favorite program to solve $Ax = b$. Then check the residual $\|Ax - b\|_2$ to at least 16 decimal places. What do you find? Compare your answer to a) and b).

4) a) Let $n \in \mathbb{N}$ and let $A \in \mathbb{C}^{m \times m}$. Show that if λ is an eigenvalue of A, then λ^n is an eigenvalue of A^n. Some hints: use the definition of an eigenvalue to get a nonzero vector v with $Av = \lambda v$. Then apply A, A^2, A^3, etc to both sides and note a pattern.

b) A matrix $A \in \mathbb{C}^{m \times m}$ is *nilpotent* if there exists $n \in \mathbb{N}$ with $A^n = 0_{m \times m}$. Show that if A is nilpotent then 0 is the only eigenvalue of A. Some hints: suppose $A^n = 0_{m \times m}$ and λ is a nonzero eigenvalue of A. Let v be a nonzero vector with $Av = \lambda v$. Then quote part a).

c) Problem 24.4 a) in the text, only do the direction "$\lim_{n \to \infty} \|A^n\| = 0 \Rightarrow \rho(A) < 1$." Some hints: By problem 3.2, $\rho(A) \leq \|A\|$ for any norm. Let λ

be an eigenvalue of A with a corresponding eigenvector v of unit length. We have $|\lambda|^n = \|A^n v\| \leq \|A^n\|$ by definition. Now take a limit.