Math 300 Assignment 2

Due Thursday, September 28

1) (#6, Section 2.1) Construct a truth table for each of the following statemtents:

a) $P \rightarrow Q$

c) $\neg P \rightarrow \neg Q$

b) $Q \rightarrow P$

d) $\neg Q \rightarrow \neg P$

Do any of these statements have the same truth table?

2) (#3, Section 2.2) Write a useful negation of each of the following statements. Do not leave a negation as a prefix of a statement. For example, we would write the negation of "I will play golf and I will mow the lawn" as "I will not play golf or I will not mow the lawn."

(a) We will win the first game and we will win the second game.

(b) They will lose the first game or they will lose the second game.

(c) If you mow the lawn, then I will pay you $20.

(d) If we do not win the first game, then we will not play a second game.

(e) I will wash the car or I will mow the lawn.

(f) If you graduate from college, then you will get a job or you will go to graduate school.

(g) If I play tennis, then I will wash the car or I will do the dishes.

(h) If you clean your room or do the dishes, then you can go to see a movie.

(i) It is warm outside and if it does not rain, then I will play golf.

3) Consider the following sets:

$A = \{2, 7, \text{cat,plutonium,chalk}\}, B = \{\text{red,blue},3,4,7,\text{chalk}\}, C = \{2, 7\},$

and let $X = A \cup B \cup C$.

For which $x \in X$ are the following conditional statements true? Give a list of x for each part.

(a) $x \in A \Rightarrow x \in C$.

(b) $(x \in C$ or $x \in A) \Rightarrow x \in A$

(c) $x \in C \Rightarrow x \in A$

(d) $(x \in A$ and $x \in B) \Rightarrow x \in C$

4) (#5, Section 2.3) Use set builder notation to specify the following sets:

(a) The set of all integers greater than or equal to 5.

(b) The set of all even integers.

(c) The set of all positive rational numbers.

(d) The set of all real numbers greater than 1 and less than 7.

(e) The set of all real numbers whose square is greater than 10.

5) Let x, y be in $\mathbb{Q}$. Prove that $x + y$ is also in $\mathbb{Q}$.

6) We now come to an interesting experiment conducted by P.C.Wason on understanding. (A lot of people get this wrong.) Suppose that students were told that each card has one side that contains a letter and the other side contains a number. The students might be shown the cards: (need a box around each one)

A		J		3		8

The task was to decide whether the rule "If a card has a vowel on one side, then it has an even number on the other side" was true or not. The students had to turn over only the cards which had to be turned over to judge the correctness of the rule. Which cards in the above example would you turn over and which would you not? Justify your answers.

7) Prove that if X is a set with n elements, $\mathcal{P}(X)$ has 2^n elements, so that

$$\operatorname{card}(\mathcal{P}(X)) > \operatorname{card}(X)$$