Math 300 Assignment 3

Due Tuesday, October 10

1) (#15, Section 3.1) Let h and k be real numbers and let r be a positive number. The equation for a circle whose center is at the point (h, k) and whose radius is r is

$$(x-h)^2 + (y-k)^2 = r^2.$$

We also know that if a and b are real numbers, then

- The point (a, b) is inside the circle if $(x-h)^2 + (y-k)^2 < r^2$.
- The point (a, b) is on the circle if $(x-h)^2 + (y-k)^2 = r^2$.
- The point (a, b) is outside the circle if $(x-h)^2 + (y-k)^2 > r^2$.

Prove that all points on or inside the circle whose equation is $(x-1)^2 + (y-2)^2 = 4$ are inside the circle whose equation is $x^2 + y^2 = 26$.

2) (#9, Section 3.2) Is the following proposition true or false? Explain.

For each positive real number x, if x is irrational, then $\sqrt{x}$ is irrational.

3) Let V be a vector space over $\mathbb{R}$ and let W be a subspace of V. Define, for $x, y \in V$,

$$x \sim y \text{ if } x - y \in W.$$

Show that "$\sim$" is an equivalence relation on V.

4) (#11, Section 7.2) Let U be a finite, nonempty set and let $\mathcal{P}(U)$ be the power set of U. That is, $\mathcal{P}(U)$ is the set of all subsets of U. Define the relation "$\sim$" on $\mathcal{P}(U)$ as follows: For $A, B \in \mathcal{P}(U)$, $A \sim B$ if and only if $A \cap B = \emptyset$. That is, the ordered pair (A, B) is in the relation "$\sim$" if and only if A and B are disjoint.

Is the relation "$\sim$" an equivalence relation $\mathcal{P}(U)$? If not, is it reflexive, symmetric, or transitive? Justify all conclusions.

5) Define a relation on $\mathbb{Z}$ as $a \sim b$ if and only if $4a + 3b$ is even. Briefly justify that "$\sim$" is **NOT** an equivalence relation on $\mathbb{Z}$.

6) Let $X = \{0, 1, 2, 3, 5\}$ and $Y = \{1, 2, 3\}$.

(a) How many ordered pairs are in $X \times Y$ and $Y \times X$ respectively?

(b) How many ordered triples are in $Y \times X \times Y$?

(c) List the elements of the set $\{(a, b, c) \in X \times Y \times X \mid a < b < c\}$.