Math 300 Assignment 4

Due Tuesday, October 31

1) (#5, Section 3.3) Prove the following propositions:

(a) For all real numbers x and y, if x is rational and y is irrational, then $x + y$ is irrational.

(b) For all nonzero real numbers x and y, if x is rational and y is irrational, then $\frac{x}{y}$ is irrational.

2) (#11, Section 3.4) Let a be a positive real number. In Part (1) of Theorem 3.25, we proved that for each real number x, $|x| < a$ if and only if $-a < x < a$. It is important to realize that the sentence $-a < x < a$ is actually the conjunction of two inequalities. That is, $-a < x < a$ means that $-a < x$ and $x < a$.

(a) Complete the following statement: For each real number x, $|x| \geq a$ if and only if...

(b) Prove that for each real number x, $|x| \leq a$ if and only if $-a \leq x \leq a$.

(c) Complete the following statement: For each real number x, $|x| > a$ if and only if...

3) (#8, Section 6.2) Let $g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ be defined by $g(m, n) = (2m, m - n)$.

(a) Calculate $g(3, 5)$ and $g(-1, 4)$.

(b) Determine all the preimages of $(0, 0)$. That is, find all $(m, n) \in \mathbb{Z} \times \mathbb{Z}$ such that $g(m, n) = (0, 0)$.

(c) Determine the set of all preimages of $(8, -3)$.

(d) Determine the set of all preimages of $(1, 1)$.

(e) Is the following proposition true or false? Justify your conclusion.

For each $(s, t) \in \mathbb{Z} \times \mathbb{Z}$, there exists an $(m, n) \in \mathbb{Z} \times \mathbb{Z}$ such that $g(m, n) = (s, t)$.

4) Let f be a real-valued function of a real variable and let $a, x \in \mathbb{R}$. Negate the following sentence: For every real number $\varepsilon > 0$, there is a real number $\delta > 0$ such that

$$|f(x) - f(a)| < \varepsilon$$

whenever $|x - a| < \delta$.

5) List all possible different functions $f : \{a, b, c\} \to \{1, 2\}$.

6) Prove that for $n \in \mathbb{N}$, $\sqrt{n} \in \mathbb{Q}$ if and only if $n = m^2$ for some $m \in \mathbb{N}$.