# Math 300 Assignment 7

**Due Tuesday, December 12**

**1)** (#6, Section 6.4) Prove part (1) of Theorem 6.20.

Let $A, B$, and $C$ be nonempty sets and let $f : A \to B$ and $g : B \to C$. If $f$ and $g$ are both injections, then $g \circ f$ is an injection.

**2)** Let $S$ be a set and define an equivalence relation on $\mathcal{P}(S)$ by

$$S \sim T \text{ if } \operatorname{card}(S) = \operatorname{card}(T).$$

Prove that "$\sim$" is an equivalence relation.

**3)** (#2, Section 8.1)

(a) Let $a \in \mathbb{Z}$ and let $k \in \mathbb{Z}$ with $k \neq 0$. Prove that if $k \mid a$ and $k \mid (a+1)$, then $k \mid 1$, and hence $k = \pm 1$.

(b) Let $a \in \mathbb{Z}$. Find the greatest common divisor of the consecutive integers $a$ and $a+1$. That is, determine $\gcd(a, a+1)$.

**4)** a) Let $n, k \in \mathbb{N}$. prove that $\gcd(n, n+k) = \gcd(n, k)$.

b) (Extra Credit) Let $m, n \in \mathbb{N}$. Prove that $\gcd(f_n, f_m) = f_{\gcd(n,m)}$ where $f_k$ is the $k^{th}$ Fibonacci number for $k \in \mathbb{N}$.

**5)**

(a) Show that, for any $n \in \mathbb{N}$, $\times_{i=1}^{n} \mathbb{N}$ is countable.

(b) Explain how part (a) proves the following statement: a
finite direct product of countable sets is countable.

(c) (Extra Credit) Prove that the countable union of countable sets is countable.

**6)** Define $f : \mathbb{Z}_2 \times \mathbb{Z}_3 \to \mathbb{Z}_6$ by

$$f([x]_2, [y]_3) = [2y - 3x]_6$$

(a) Show that $f$ is bijective.

(b) Find the inverse function of $f$, and carefully justify that this is the inverse.