Math 300 Midterm 1

Thursday, October 19th

The even-numbered problems are definitions meant to aid you in the subsequent odd-numbered problem. Use them wisely.

1) Let P, Q, and R be statements.

a) Negate the compound statement $P \Rightarrow (Q \Rightarrow R)$.

b) Show that the following compound statements are logically equivalent:

$$P \Rightarrow (Q \Rightarrow R) \text{ and } (P \wedge Q) \Rightarrow R.$$

2) a) Define what it means for a real number x to be a rational number.

b) Define an equivalence relation "$\sim$" on a set S (alternatively, you may define an equivalence relation as a subset of $S \times S$).

3) Define "$\sim$" on $\mathbb{R}$ by

$$x \sim y \text{ if } x - y \in \mathbb{Q}.$$

Prove that "$\sim$" is an equivalence relation. You may assume that products and sums of rational numbers are rational.

4) Let S be a universal set. Let $A, B \subseteq S$.

a) Define the intersection of A and B.

b) Define $S \backslash A$.

c) Define the power set $\mathcal{P}(S)$.

5) Let S be a set and let $\emptyset \neq A \subset S$. Show that $T \in \mathcal{P}(S)$ if and only if there exist $T_1 \in \mathcal{P}(A)$ and $T_2 \in \mathcal{P}(S \backslash A)$ with $T = T_1 \cup T_2$.