Math 300 Midterm 2

Tuesday, November 21st

The even-numbered problems are definitions meant to aid you in the subsequent odd-numbered problem. Use them wisely.

1) Prove that $3^n > n + 1$ for all $n \in \mathbb{N}$.

2) Let S and T be sets. Let $\phi : S \to T$ be a function.

a) What does it mean for ϕ to be injective?

b) What does it mean for ϕ to be surjective?

c) What does it mean for ϕ to be bijective?

3) Let $M_2(\mathbb{R})$ denote the 2×2 matrices with real entries. Define

$$\phi : M_2(\mathbb{R}) \to M_2(\mathbb{R})$$

by

$$\phi\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} -a & c \\ b & -d \end{bmatrix}$$

Prove that ϕ is a bijection.

4) Let S be a universal set. Let $A, B \subseteq S$. State DeMorgan's Laws:

a) $(A \cap B)^c =$ ______________

b) $(A \cup B)^c =$ ______________

5) Let S and T be universal sets and let $A \subseteq S$, $B \subseteq T$. Prove that

$$A \times B = (A \times B^c)^c \cap (A^c \times B)^c \cap (A^c \times B^c)^c.$$