Announcements

1) HW 4 due next Tuesday

2) Reading for Thursday:

   Section 4.2

Mathematical Induction

(Chapter 4)

A bootstrap procedure where, to prove that a statement $P(n)$ is true for all values of $n \in \mathbb{N}$, you:

1) Prove that $P(1)$ is true.
2) Use the fact that $P(1)$ is true to show $P(2)$ is true.
3) Use the fact that $P(2)$ is true to show $P(3)$ is true

Keep on going!

Shortcut

You want to show $P(n)$ is true for all $n \in \mathbb{N}$. You then

1) Show that $P(1)$ is true.

2) Assume $P(n)$ is true.

   Use this to prove $P(n+1)$ is true. $(n \geq 1)$

Example 1: $\displaystyle\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \quad \forall n \in \mathbb{N}.$

Prove by induction!

$P(n)$ is the statement

" $\displaystyle\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ ".

$P(1)$ $n=1$. The left-hand side

of the equality is

$$\sum_{k=1}^{1} k = 1$$

The right hand side is

$$\frac{1(1+1)}{2} = \frac{1 \cdot 2}{2} = 1 \checkmark$$

So $P(1)$ is true.

Suppose $P(n)$ is true. We want to show $P(n+1)$ is true.

$P(n)$ is the statement

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$P(n+1)$ is the statement

$$\sum_{k=1}^{n+1} k = \frac{(n+1)(n+1+1)}{2} = \frac{(n+1)(n+2)}{2}$$

P(n) is the statement

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

P(n+1) is the statement

$$\sum_{k=1}^{n+1} k = \frac{(n+1)(n+1+1)}{2} = \frac{(n+1)(n+2)}{2}$$

We know P(n) is true,

So $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

To get P(n+1), add (n+1) to both sides

Then

$$\sum_{k=1}^{n} k + (n+1) = \frac{n(n+1)}{2} + (n+1)$$

$$= \sum_{k=1}^{n+1} k \qquad = \frac{n(n+1)}{2} + \frac{2n+2}{2}$$

$$= \frac{n^2 + 3n + 2}{2}$$

$$= \frac{(n+1)(n+2)}{2}$$

Therefore, the statement is true by induction.

The Principle of Mathematical Induction

Let $T \subseteq \mathbb{N}$ be a nonempty subset of $\mathbb{N}$. Suppose

1) $1 \in T$

2) If $n \in T$, then $(n+1) \in T$.

Then $T = \mathbb{N}$

This is behind every inductive proof!