Announcements

1) HW 4 due Tuesday

2) Colloquium today 2:30
CB 2046 Ravi Ramasami
"Spectrum of the Kohn Laplacian on the Rossi Sphere"

3) Halloween Tuesday

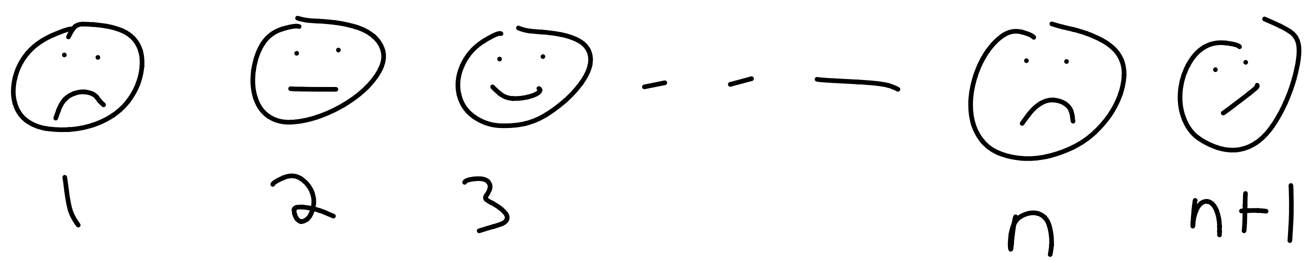
Costumes = Candy

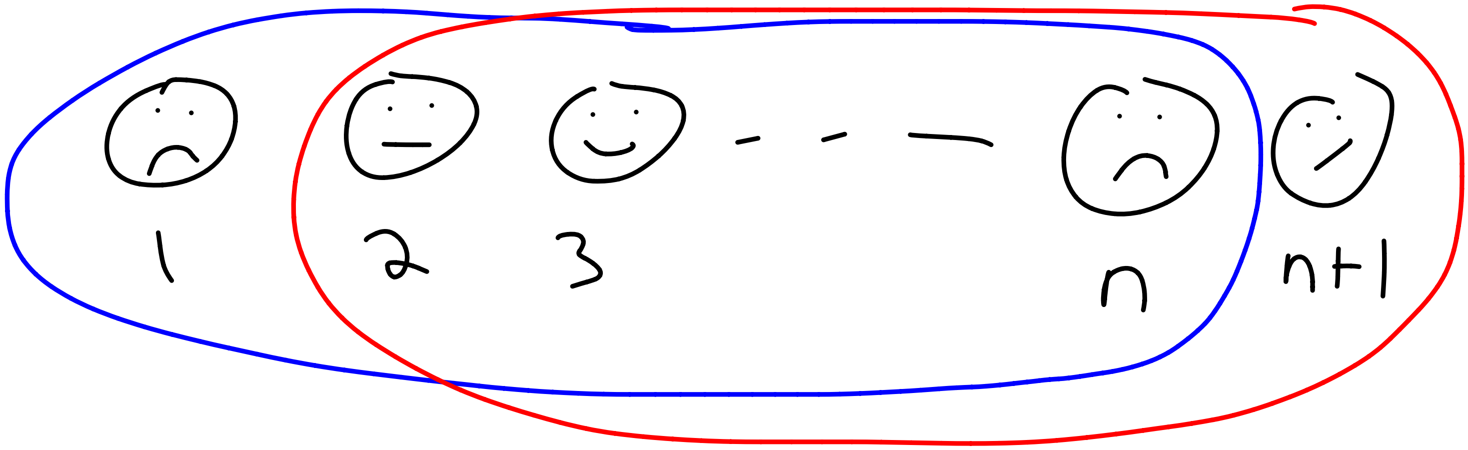
Example 1: All babies have blue eyes.

$n=1$ step. My son has blue eyes.

Done

Assume that n babies have blue eyes. Consider any collection of $n+1$ babies





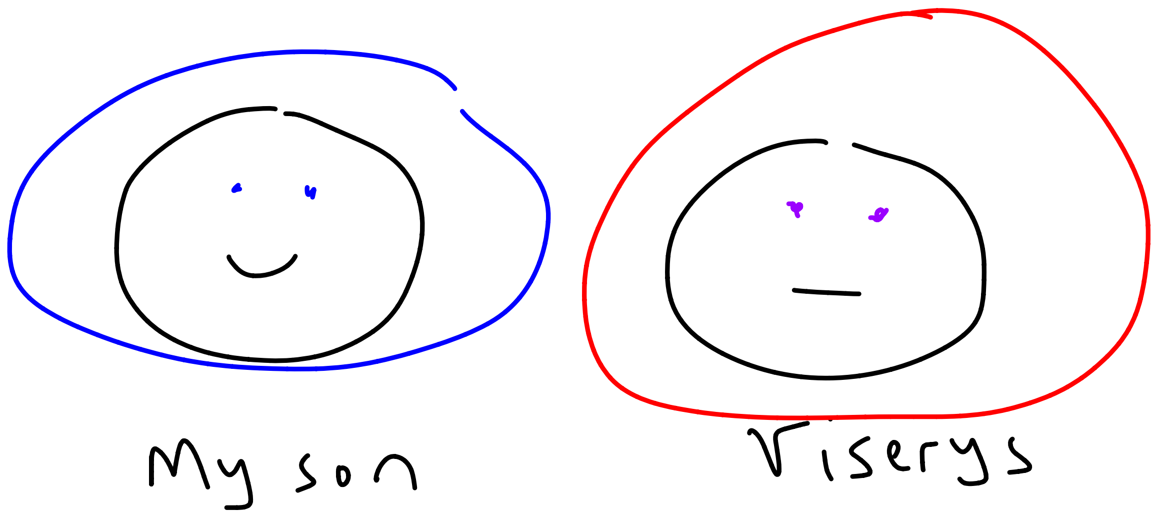
By induction, all babies in the blue circle have blue eyes.

Again by induction, all babies in the red circle have blue eyes.

Therefore, all babies have blue eyes by induction

False fails for n=2

2 babies



There is no way to conclude another baby has blue eyes by just knowing my son has blue eyes

Example 2: If $a_1, a_2, \ldots, a_{2^n} \in \mathbb{R}$,

$a_i \geq 0 \; \forall \; 1 \leq i \leq 2^n$,

then

$$\underbrace{(a_1 a_2 \cdots a_{2^n})^{1/2^n}}_{\text{geometric average}} \leq \underbrace{\frac{\sum_{k=1}^{2^n} a_k}{2^n}}_{\text{arithmetic average}}.$$

This is called the arithmetic-geometric mean inequality.

n=1 $\quad (a_1 \cdot a_2)^{\frac{1}{2}} \overset{?}{\leq} \dfrac{a_1 + a_2}{2}$

Square both sides.

$$a_1 a_2 \overset{?}{\leq} \frac{(a_1+a_2)^2}{4}$$

$$a_1 a_2 \overset{?}{\leq} \frac{a_1^2 + 2a_1a_2 + a_2^2}{4}$$

multiply both sides by 4.

$$4a_1a_2 \overset{?}{\leq} a_1^2 + 2a_1a_2 + a_2^2$$

subtract $4a_1a_2$ from both sides

$$0 \overset{?}{\leq} a_1^2 - 2a_1a_2 + a_2^2$$

$$0 \overset{?}{\leq} (a_1 - a_2)^2 \checkmark$$

True

Inductive Step: Assume

$$(a_1 a_2 \cdots a_{2^n})^{1/2^n} \le \frac{\sum_{k=1}^{2^n} a_k}{2^n}$$

Show

$$(a_1 a_2 \cdots a_{2^n} a_{2^{n+1}})^{\frac{1}{2^{n+1}}} \le \frac{\sum_{k=1}^{2^{n+1}} a_k}{2^{n+1}}$$

$$= \left(\underbrace{\sqrt{(a_1 a_2)}}_{b_1}\underbrace{\sqrt{(a_3 a_4)}}_{b_2}\underbrace{\sqrt{(a_5 a_6)}}_{b_3} \cdots \underbrace{\sqrt{(a_{2^n} a_{2^{n+1}})}}_{b_{2^n}}\right)^{\frac{1}{2^n}}$$

$$= (b_1 \cdots b_{2^n})^{1/2^n}$$

$$\le \left(\frac{\sum_{k=1}^{2^n} b_k}{2^n}\right)^{1/2}$$

$$= \frac{\sum_{k=1}^{2^n} b_k}{2^n}$$

$$b_1 = \sqrt{a_1 a_2} \leq \frac{a_1 + a_2}{2}$$

$$b_2 = \sqrt{a_3 a_4} \leq \frac{a_3 + a_4}{2}$$

$$b_k = \sqrt{a_{2k-1} a_{2k}}$$

$$\leq \frac{a_{2k-1} + a_{2k}}{2}$$

$$\frac{\sum_{k=1}^{2^n} b_k}{2^n} \le \frac{\sum_{k=1}^{2^n} \left(\frac{a_{2k-1}+a_{2k}}{2}\right)}{2^n}$$

$$= \frac{\sum_{k=1}^{2^n} a_{2k-1}+a_{2k}}{2^{n+1}}$$

$$= \frac{\sum_{k=1}^{2^{n+1}} a_k}{2^{n+1}}$$