Announcements

1) HW due Thursday

2) HW 2 up later today

Logical Operations on Statements

(Section 2.1)

Start with statements P and Q.

The disjunction of P and Q is the statement "P or Q", written "$P \vee Q$" where "or" is inclusive.

The conjunction of P and Q is the statement "P and Q", written "$P \wedge Q$".

The negation of P, written "$\neg P$", is the negation of the statement P (can also use "not P")

The implication "$P \to Q$" is the conditional statement "if P, then Q" (can also use "$P \Rightarrow Q$")

Example 1: Let P be the sentence "I drive a Honda" and Q be the sentence "Today is a day of the week".

Negations: $\neg P$ = I don't drive a Honda.

$\neg Q$ = Today is not a day of the week.

Implication: $P \Rightarrow Q$ If I drive a Honda, then today is a day of the week.

Fun with truth tables

For a statement composed out many sub-statements, you can use a truth table to determine the circumstances in which the compound statement is true based off of the truth value of its components.

Tautologies

A **tautology** is a compound sentence whose truth value is independent of the truth value of its components.

For example:

$$P \vee (\neg P)$$

is true regardless of whether P is true

Logically Equivalent Statements

(Section 2.1)

Two compound statements are **logically equivalent** if they have the same truth tables.

Theorem: (DeMorgan's Laws)

Let P and Q be statements.

Then

1) $\neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q)$

2) $\neg(P \wedge Q) \equiv (\neg P) \vee (\neg Q)$

where "$\equiv$" means logical equivalence.

proof: 1) Need two truth tables!

P	T	T	F	F
Q	T	F	T	F
$P \vee Q$	T	T	T	F
$\neg(P \vee Q)$	F	F	F	T

P	T	T	F	F
Q	T	F	T	F
$\neg P$	F	F	T	T
$\neg Q$	F	T	F	T
$(\neg P) \wedge (\neg Q)$	F	F	F	T

Bottom Rows match

for the associated values of P & Q, so the statements are logically equivalent.

2) Similar

$\square$

Converse and Contrapositive

Given two statements P and Q and the conditional "$P \Rightarrow Q$", the converse of "$P \Rightarrow Q$" is the statement "$Q \Rightarrow P$". The contrapositive of "$P \Rightarrow Q$" is the statement "$(\neg Q) \Rightarrow (\neg P)$".

The contrapositive is logically equivalent to the conditional, but the converse may not be!

Example 2 Consider the statement

"If p is a prime number, then p is odd."

The converse of this statement is

"If p is odd, then p is a prime number."

The contrapositive of this statement is "If p is not odd, then p is not a prime number."

Negations of Conditionals

The negation of the statement

"$P \Rightarrow Q$" is the statement

"$(\neg P) \wedge Q$" (remember when

$P \Rightarrow Q$ is false!)

Example 3: Negate the statement

"If M is a square matrix, then zero is an eigenvalue of M."

$P =$ "M is a square matrix"

$Q =$ "zero is an eigenvalue of M", so the negation is

"M is a square matrix and zero is not an eigenvalue of M".