

# Announcements

1) HW due Thursday

2) HW 2 up later today

# Logical Operations on Statements

(Section 2.1)

Start with statements  $P$  and  $Q$ .

The **disjunction** of  $P$  and  $Q$  is the statement " $P$  or  $Q$ ",

written " $P \vee Q$ " where

"or" is inclusive.

The **conjunction** of  $P$  and  $Q$

is the statement " $\bar{P}$  and  $Q$ ",

written " $\bar{P} \wedge Q$ ".

The negation of  $P$ , written " $\neg P$ ", is the negation of the statement  $P$  (can also use "not  $P$ ")

The implication " $P \rightarrow Q$ " is the conditional statement "if  $P$ , then  $Q$ " (can also use " $P \Rightarrow Q$ ")

Example 1: Let  $P$  be the sentence

"I drive a Honda" and

$Q$  be the sentence "Today is a day of the week."

Negations:  $\neg P =$  I don't drive a Honda.

$\neg Q =$  Today is not a day of the week.

Implication:  $P \Rightarrow Q$  If I drive a Honda, then today is a day of the week.

## Fun with truth tables

For a statement composed out of many sub-statements, you can use a truth table to determine the circumstances in which the compound statement is true based off of the truth value of its components.

## Tautologies

A **tautology** is a compound sentence whose truth value is independent of the truth value of its components.

For example:

$$P \vee (\neg P)$$

is true regardless of whether

$P$  is true

# Logically Equivalent Statements

(Section 2.1)

Two compound statements are logically equivalent if they have the same truth tables.

Theorem: (DeMorgan's Laws)

Let  $P$  and  $Q$  be statements.

Then

$$1) \neg (P \vee Q) = (\neg P) \wedge (\neg Q)$$

$$2) \neg (P \wedge Q) = (\neg P) \vee (\neg Q)$$

where " $=$ " means logical equivalence.

proof: 1) Need two truth tables!



P	T	T	F	F
Q	T	F	T	F
$P \vee Q$	T	T	T	F
$\neg(P \vee Q)$	F	F	F	T



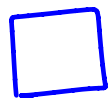
P	T	T	F	F
Q	T	F	T	F
$\neg P$	F	F	T	T
$\neg Q$	F	T	F	T
$(\neg P) \wedge (\neg Q)$	F	F	F	T



Bottom Rows match

for the associated values  
of  $P \downarrow Q$ , so the statements  
are logically equivalent.

2) Similar



## Converse and Contrapositive

Given two statements  $P$  and  $Q$  and the conditional " $P \Rightarrow Q$ ", the **converse** of " $P \Rightarrow Q$ " is the statement " $Q \Rightarrow P$ ". The **contrapositive** of " $P \Rightarrow Q$ " is the statement " $(\neg Q) \Rightarrow (\neg P)$ ".

The contrapositive is logically equivalent to the conditional, but the converse may not be!

Example 2 Consider the statement

"If  $p$  is a prime number,  
then  $p$  is odd."

The converse of this statement is

"If  $p$  is odd, then  $p$  is a  
prime number."

The contrapositive of this statement  
is "If  $p$  is not odd, then  
 $p$  is not a prime number."

## Negations of Conditionals

The negation of the statement

" $P \Rightarrow Q$ " is the statement

" $(\neg P) \wedge Q$ " (remember when

$P \Rightarrow Q$  is false!)

Example 3 : Negate the statement

"IF  $M$  is a square matrix,  
then zero is an eigenvalue of  $M$ ."

$P =$  " $M$  is a square matrix"

$Q =$  "zero is an eigenvalue of  $M$ ", so

the negation is

" $M$  is a square matrix and  
zero is not an eigenvalue of  $M$ ."