Math 300 In-Class Worksheet 10: Induction

1) For all $n \in \mathbb{N}$ show that

$$\frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \text{.........} + \frac{1}{(n+1)(n+2)} = \frac{n}{2(n+2)}$$

2) Prove the following distributive property for subsets A and B_i, $1 \leq i \leq n$, of a universal set U:

$$A \cap \left(\cup_{i=1}^{n} B_i \right) = \cup_{i=1}^{n} (A \cap B_i).$$

3) Let $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2 e^x$. Show that the nth derivative of f is equal to $(x^2 + 2nx + n(n-1))e^x$. You may assume knowledge of the product rule, as well as the derivatives of x^2 and e^x.

4) Assume that the set, $S \subseteq \mathbb{N}$, is any set that has the two properties:

(i) $7 \in S$

(ii) If $x \in S$, then $x + 4 \in S$.

Use induction to prove that:

$$T = \{4k + 3 \mid k \in \mathbb{N}\} \subseteq S.$$