Math 300 In-Class Worksheet 11: Injections, Surjections, and Bijections

1) For each of the following functions, say whether or not it is injective, surjective, or bijective and justify your response.
Hint 1: *you may find it helpful to complete the square... if you forgot what that is, you can look it up.*
Hint 2: *after you complete the square, it could be very helpful to sketch a graph of each function, paying careful attention to the domain and co-domain in each case.*

(a) $f : \mathbb{R} \to \mathbb{R}$, with the assignment rule $f(x) = x^2 + 6x + 5$.

(b) $g : \mathbb{R} \to [-4, \infty)$, with the assignment rule $g(x) = x^2 + 6x + 5$.

(c) $h : [-3, \infty) \to \mathbb{R}$, with the assignment rule $h(x) = x^2 + 6x + 5$.

(d) $i : [-3, \infty) \to [-4, \infty)$, with the assignment rule $i(x) = x^2 + 6x + 5$.

2) Let $a_1 = 5$, $a_{n+1} = \sqrt{a_n + 25}$. Prove that $a_n < 5.8$ for all $n \geq 1$. You may use a calculator to check what are the values of some square roots.

3) Suppose $f : A \to B$ and $g : X \to Y$ are bijective functions. Define a new function $h : A \times X \to B \times Y$ by $h(a, x) = (f(a), g(x))$. Prove that h is bijective.

4) (The littlest equivalence relation) Let A be any set and define the relation " $\sim$ " on A by

$$x \sim y \text{ if and only if } x = y$$

for all $x, y \in A$.

a) If $x \in A$, determine card($[x]$)

a) Prove that " $\sim$ " is an equivalence relation.

c) Show that the function $\phi : A \to A/\sim$ given by $\phi(x) = [x]$ is a bijection.