Math 300 In-Class Worksheet 12: Strong Induction and Recursion

1) (# 12, Section 4.2) Prove that for each natural number n, any set with n elements has $n(n-1)/2$ two-element subsets.

2) For which natural numbers n do there exist nonnegative integers x and y such that $n = 4x + 5y$? Justify your conclusion.

3) Let $a_1 = 5$, $a_{n+1} = \sqrt{a_n + 25}$. Prove that $a_n < 5.8$ for all $n \geq 1$. You may use a calculator to check what are the values of some square roots.

4) Let $(f_n)_{n=1}^{\infty}$ be the Fibonacci numbers. Show that for all $k \in \mathbb{N}$,

$$f_{n+k} = f_n f_{k+1} + f_{n-1} f_k.$$

This may be helpful on a future homework!