Math 300 In-Class Worksheet 13: More on Functions and Sets

1) (#2, Section 5.3) Let A, B, and C be subsets of some universal set U. As part of Theorem 5.18, we proved one of the distributive laws. Prove the other one. That is, prove that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

2) (#8, Section 5.3)

(a) Draw two general Venn diagrams for the sets A, B, and C. On one, shade the region that represents $A-(B-C)$, and on the other, shade the region that represents $(A-B)\cup(A-C^c)$. Based on the Venn diagrams, make a conjecture about the relationship between the sets $A-(B-C)$ and $(A-B)\cup(A-C^c)$. (Are the two sets equal? If not, is one of the sets a subset of the other set?)

(b) Prove the conjecture from Exercise (8a).

3) Define $D : \mathbb{P}_4 \to \mathbb{P}_3$ by

$$D(p(x)) = p'(x)$$

where p' is the derivative of p.

(a) Is I an injective function? Justify your answer.

(b) Is I a surjective function? Justify your answer.

4) Let $.a_1a_2a_3\ldots$ denote the usual (base-10) decimal expansion of an element in $[0,1]$. Consider $f:[0,1]\to[0,1]$ by

$$f(.a_1a_2a_3\ldots) = .a_2a_4a_6\ldots$$

Assuming f is well-defined,

(a) Is I an injective function? Justify your answer.

(b) Is I a surjective function? Justify your answer.