Math 300 In-Class Worksheet 14: Composition and Inverses of Functions

1) Show that the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by the formula $f(x, y) = ((x^2 + 1)y, x^3)$ is bijective. Then find its inverse. Carefully justify that your answer does indeed yield the inverse function.

2) (#7, Section 6.4) For each of the following, give an example of functions $f : A \to B$ and $g : B \to C$ that satisfy the stated condition, or explain why no such example exists.

(a) The function f is a surjection, but the function $g \circ f$ is not a surjection.

(b) The function f is an injection, but the function $g \circ f$ is not an injection.

(c) The function g is a surjection, but the function $g \circ f$ is not a surjection.

(d) The function g is an injection, but the function $g \circ f$ is not an injection.

(e) The function f is not a surjection, but the function $g \circ f$ is a surjection.

(f) The function f is not an injection, but the function $g \circ f$ is an injection.

(g) The function g is not a surjection, but the function $g \circ f$ is a surjection.

(h) The function g is not an injection, but the function $g \circ f$ is an injection.

3) Define $\phi : \mathbb{Z}_2 \times \mathbb{Z}_2 \to \mathbb{Z}_4$ by

$$\phi([a]_2, [b]_2) = [2a + b]_4$$

(a) Show that ϕ is bijective.

(b) Find the inverse function of ϕ, and carefully justify that this is the inverse.

4) Prove that function composition is associative.