Math 300 In-Class Worksheet 15: Greatest Common Divisors

1) (#3, Section 8.1)

(i) Let $a \in \mathbb{Z}$ and let $k \in \mathbb{Z}$ with $k \neq 0$. Prove that if $k \mid a$ and $k \mid (a+2)$, then $k \mid 2$.

(ii) Let $a \in \mathbb{Z}$. What conclusions can be made about the greatest common divisor of a and $a+2$?

2) Prove or disprove: if a divides c and b divides c and $\gcd(a, b) = d$, then ab divides cd.

3) Prove or disprove: $m^3 - m$ is divisible by 6 for all $m \in \mathbb{N}$.

4) Prove or disprove: there exist infinitely many pairs of integers m and n with $m + n = 100$ and $\gcd(m, n) = 5$.