Math 300 In-Class Worksheet 16: Prime Factorizations and Function Rules

1) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 0, & x \in \mathbb{Q} \\ x, & x \notin \mathbb{Q}. \end{cases}$$

a) Is f a bijection? That is, does f^{-1} define a function?

b) Compute $f^{-1}(\{2\})$, $f^{-1}(\{\pi\})$, and $f^{-1}([0, 1])$.

c) Find the largest domain on which f is injective.

2) (#5, Section 6.6) Prove part (2) of Theorem 6.34:

Let $f : S \to T$ be a function and let A and B be subsets of S. Then $f(A \cup B) = f(A) \cup f(B)$.

3)

4) (#18, Section 8.2) Prove the following proposition

For all natural numbers m and n, if m and n are twin primes other than the pair 3 and 5, then 36 divides $mn + 1$ and $mn + 1$ is a perfect square.

Hint: Look at several examples of twin primes. What do you notice about the number that is between the two twin primes? Set up cases based on this observation.

5) fun