

## Math 300 In-Class Worksheet 2: Statements

1) Let's go back to the worksheet from last time. Armed with a little more knowledge, which of the following sentences are statements? If they are, can you decide whether they are true or false?

- (a) Pythagoras was vicious.
- (b) The square root of a negative number is a negative number.
- (c)  $4x^3 - 9x^2 + 3x - 25 = 0$
- (d) Socrates was a very wise man.
- (e) There is a blackboard in every classroom in the CASL Building.
- (f) Blackboards are superior to whiteboards.
- (g) If  $p$  is prime, its only divisors are 1 and  $p$ .
- (h) For all angles  $\theta$ ,  $0 \leq \theta \leq \pi$ ,  $\sin \theta$  is positive.

2) Identify the hypothesis and the conclusion for each of the following conditional statements.

- (a) If  $n$  is a prime number, then  $n^2$  has three positive factors.
- (b) If  $a$  is an irrational number and  $b$  is an irrational number, then  $a \cdot b$  is an irrational number.
- (c) If  $p$  is a prime number, then  $p = 2$  or  $p$  is an odd number.
- (d) If  $p$  is a prime number and  $p \neq 2$ , then  $p$  is an odd number.
- (e) If  $p \neq 2$  and  $P$  is an even number, then  $p$  is not prime.

3) (#6, Section 1.1) Let  $a, b, c \in \mathbb{R}$ . Consider the following theorem:

**Theorem 0.1.** *If  $f$  is a quadratic function of the form  $f(x) = ax^2 + bx + c$  and  $a < 0$ , then the function  $f$  has a maximum value with  $x = -\frac{b}{2a}$ .*

Using **only** this theorem, what can be concluded about the functions given by the following formulas?

- a)  $g(x) = -8x^2 + 5x - 2$       b)  $h(x) = -\frac{1}{3}x^3 + 3x$   
c)  $k(x) = 8x^2 - 5x - 7$       d)  $F(x) = -x^4 + x^2 + 9$

- 4) Let's say you want to **prove** the theorem from the previous question.
- a) Would it be enough to check the theorem on 5 examples?
  - b) Would it be enough to check the theorem on 5555 examples?
  - c) How many examples would you have to check to prove this theorem?
  - d) Now try to prove the theorem.

5) Walking Boss Godfrey is playing a game with three inmates in his prison. He puts blindfolds on all three prisoners and tells them he has five hats, three of which are red and two of which are blue. He puts a hat on each prisoner and hides the remainder. He then informs the inmates that he will remove their blindfolds one at a time, and if any one of them can deduce the color of their own hat by only looking at the other two prisoners, that person can go free. If they guess wrong or tell another prisoner the color of their hat, though, it's solitary confinement for a week! There is no penalty for not guessing besides staying in jail.

Boss Godfrey takes the blindfold off the first prisoner, who says he can't determine the color of his own hat. The second prisoner makes the same declaration after his blindfold is removed. The third prisoner, who is Cool Hand Luke, then tells Walking Boss Godfrey the color of his own hat without even taking off his blindfold.

What color is Luke's hat, and how does he know?