

Math 300 In-Class Worksheet 5: Direct Proofs

1) Negate the following statements:

(a) $\forall n \in \mathbb{Z}$, 2 divides n .

(b) $\exists x \in \mathbb{Q}$, $x \notin \mathbb{R}$.

(c) All cars have at least one wheel.

(d) $\forall x \in \mathbb{R}$, \exists polynomial p with integer coefficients, $p(x) = 0$.

(e) $\exists x \in \mathbb{R}$ such that \forall polynomials p with integer coefficients, $p(x) = 0$.

2) Let S, T be invertible 2×2 matrices with real coefficients. Prove that ST is also invertible.

3) Show that the number of different orderings of an n element set is equal to $n!$.

4) Emperor Palpatine has captured Rebel spies whom he wishes to interrogate. However, wanting to deliver his message of order through fear to the upstart alliance, he has decided to release one of the spies, but he will only choose the spy to be released via a rather odd method: he arranges the spies in a circle and, starting with a spy labelled #1 and continuing in numerical order, recites, “Skip, cell, skip, cell, skip, cell,...”. Anyone who got “cell” goes back to their cell.

So if the Emperor has 4 prisoners, he says “Skip, cell, skip, cell,” and prisoners 2 and 4 go back to their cells, leaving 1 and 3. Then he starts where he left off (if he ended with “skip”, he starts with “cell” now, and if he ended with “cell,” he starts with “skip”) and says “Skip, cell,” so prisoner 3 goes back to their cell, leaving only prisoner 1. When there’s only one prisoner left, the Emperor releases them- but not before turning them over to Darth Vader for interrogation.

a) Is there a difference between the Emperor starting with “skip” or with “cell”? Why or why not?

b) Is there a number of total spies between 10 and 20 where if Emperor Palpatine plays his game with that many spies, spy #1 goes free? If so, what is the number? How about between 20 and 30? 30 and 40? 40 and 100?

c) Is there a number n of total spies between 10 and 20 where if Emperor Palpatine plays his game with that many spies, spy # n goes free? If so, what is the number? How about between 20 and 30? 30 and 40? 40 and 100?