**Math 300 In-Class Worksheet 6: Direct Products and Equivalence Relations**

**1)** Let the universal set $X = \{2, 3, 5\}$. Let $A = \{2, 3\}$ and $B = \{3, 5\}$. Determine the following sets and their cardinalities.

(a) $X \times X$

(b) $A \times B$

(c) $(A \times B)^c$ (Hint: notice $A \times B \subseteq X \times X$)

(d) $A \times A \times B$

(e) $B^c$

**2)** Assume that $\sim$ is an equivalence relation on $\mathbb{Z}$, and that $x \in \mathbb{Z}$. Of the following, determine the only statement which is correct, and explain, in one sentence or less, why the others are false:

(i) $[x]$ is an equation that determines $x$.

(ii) $[x]$ is an element of $\mathbb{Z}$.

(iii) $[x]$ is the collection $\{0, 1, \ldots, p-1\}$, where $p$ is a prime number such that $p > 1$.

(iv) $[x]$ is a subset of $\mathbb{Z}$.

(v) $[x]$ is a subset of $\mathbb{Z}_{\geq 0}$.

(vi) $[x]$ is the smallest integer that is greater than or equal to $x$.

**3)** Define a relation on $\mathbb{Z}$ as $x \sim y$ if and only if $2x + 3y$ is even. Briefly justify that $\sim$ is **not** an equivalence relation on $\mathbb{Z}$.

**4)** Define a relation $\sim$ on $\mathbb{Z}$ as $x \sim y$ if and only if $4 \mid (x + 3y)$. Prove $\sim$ is an equivalence relation. Describe its equivalence classes.

**5)** You're standing at three light switches at the bottom of the stairs to the attic. Each one corresponds to one of three lights in the attic, but you cannot see the lights from where you stand. You can turn the switches on and off and leave them in any position. How can you identify which switch corresponds to which light bulb if you are only allowed one trip upstairs?