Math 300 In-Class Worksheet 7: The Contrapositive

1) Write the contrapositive of the following statements.

(i) If n is even, then n^3 is even.

(ii) If f is a differentiable function, then f is continuous.

(iii) If a matrix A has an inverse, then A is a square matrix.

(iv) (Section 3.2 # 2(a)) For all integers a and b, if $a \neq 0 \pmod 6$ and $b \neq 0 \pmod 6$, then $ab \neq 0 \pmod 6$.

(v) (Section 3.2 # 3(a)) For all positive real numbers a and b, if $\sqrt{ab} \neq \dfrac{a+b}{2}$, then $a \neq b$.

2) Prove the following implication. **If** A, B, and C are sets such that

(i) $A \subseteq C$ and $B \subseteq C$,

(ii) $C = A \cup B$,

(iii) $|C| = 12$, $|B| = 5$, $|A| = 7$,

then $A \cap B = \emptyset$.

(Hint: What if you assumed that $A \cap B \neq \emptyset$? What if $|A \cap B| = 1$? In that case, how many elements could possibly be in $A \cup B$? What if $A \cap B$ has 2 or 3 elements? A Venn diagram could help you organize your thoughts.)

3) Prove that if x and y are both strictly positive real numbers, then $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$.

4) a) Michael Phelps, Ryan Lochte, and Ian Thorpe (in a comeback role) are swimming the 200m freestyle. In how many different ways can they finish first, second, and third?

b) Juan Martin del Potro, Rafael Nadal, Andy Murray, and Kei Nishikori are all in the Olympic tennis singles semifinals. In how many different ways can they finish gold, silver, and bronze (theoretically)?

c) You're ordering toppings for your Jets pizza. Each topping can be ordered repeatedly as an "extra" option. If Jets has 15 toppings and you want three of them (maybe with extras), how many choices of toppings do you have?

d) How many ways are there to order n things in n places if you can't choose the objects more than once? How many ways are there to choose n objects for n places if you can choose the objects more than once?