Math 300 In-Class Worksheet 8: Proofs by Contradiction and Cases

1) Prove that $\sqrt{26}$ is an irrational number.

2) Show that
$$\bigcap_{n\in\mathbb{Z}}[n, n + 1] = \emptyset.$$

3) Suppose $a, b, c \in \mathbb{Z}$. Prove that if $a^2 + b^2 = c^2$, then a or b is even.

4) Prove that there is no integer solution to

$$x^2 + x - 1 = 0.$$

5) Prove the triangle inequality: for all $x, y \in \mathbb{R}$,

$$|x + y| \leq |x| + |y|.$$