Linear Parameter Varying-Based Control of a Riderless Bicycle with Linear Actuators

Ronald Smith, Ziad Fawaz, Alireza Mohammad, Paul Muench, and Sridhar Lakshmanan

Department of Electrical & Computer Engineering, University of Michigan-Dearborn, Dearborn, MI 48128, USA

ABSTRACT

Riderless bicycles, which belong to the class of narrow autonomous vehicles, offer numerous potentials to improve living conditions in the smart cities of the future. Various obstacles exist in achieving full autonomy for this class of autonomous vehicles. One of these significant challenges lies within the synthesis of automatic control algorithms that provide self-balancing and maneuvering capabilities for this class of autonomous vehicles. Indeed, the nonlinear, underactuated, and non-minimum phase dynamics of riderless bicycles offer rich challenges for automatic control of these autonomous vehicles. In this paper, we report on implementing linear parameter varying (LPV)-based controllers for balancing our constructed autonomous bicycle, which is equipped with linear electric actuators for automatic steering, in the upright position. Experimental results demonstrate the effectiveness of the proposed control strategy.

Keywords: Autonomous bicycles, linear electric actuators, linear parameter varying control, gain scheduling control.

1. INTRODUCTION

Bicycles, which belong to the class of single-track vehicles, have strong off-road capabilities and high maneuverability/mobility. Consequently, autonomous bicycles offer numerous potentials for autonomous vehicle applications where there is a need for having light weight, narrow structure, and safe autonomy. Various obstacles exist in achieving full autonomy for autonomous bicycles. One of these significant challenges lies within the synthesis of automatic control algorithms that provide self-balancing and maneuvering capabilities for this class of autonomous vehicles. Indeed, when not operated by a human or automatically controlled, the bicycle becomes unstable and loses its balance. This is due to the fact that the bicycle is an underactuated dynamical system with unstable nonminimum phase roll dynamics. Automatic control of autonomous bicycles is still a rich and open problem in the area of autonomous robot/vehicle control.

Various researchers have studied the problem of designing stabilizing controllers for autonomous bicycles. A family of these controllers belong to the class of intelligent control schemes such as fuzzy control, neural network-based control, and reinforcement learning control. Another family of controllers for maintaining balance of autonomous bicycles rely on nonlinear control techniques such as sliding-mode control and input-output linearizing control. While the two aforementioned class of controllers can be used across a wide range of bicycle speeds and slip angles, their implementation is cumbersome and very few of them have been implemented on actual riderless bicycle prototypes.

A third group of self-balancing controllers, which have been successfully implemented on autonomous bicycles, rely on tools from classical and linear control. These techniques employ a linearized model of the bicycle dynamics about various operating points. Using one or more observable variables such as the bicycle speed and roll angle, it is determined what operating region the bicycle dynamics are currently evolving in and, accordingly, the proper linear controller is enabled. The framework of controlling nonlinear dynamical systems, e.g., the bicycle, using a family of linear controllers is called linear parameter varying (LPV) or gain scheduling control.

* Send correspondence to A. Mohammad.
A. Mohammad: E-mail: amohammad@umich.edu, Z. Fawaz: fawazz@umich.edu, R. Smith: ron.grant@umich.edu, P. Muench: muenchp@umich.edu, S. Lakshmanan: lakshman@umich.edu
framework. In this paper, we use the gain scheduling framework to automatically balance our constructed riderless bicycle in the upright position.

We have constructed an autonomous bicycle with linear electric actuators, which provide the bicycle with automatic steering capabilities. Using such linear actuators has several potential benefits in this application. First, having high thrust and stroke speeds, linear actuators can realize fast front wheel steering for agile operation of autonomous bicycles. Second, being relatively lightweight and having a narrow structure, linear actuators can be easily integrated into the frame of the bicycle as a replacement to the horizontal cross bar (the portion of the frame connecting the seat to the handlebar of the bicycle). Finally, using linear actuators with high backdrivability enables the autonomous bicycle to flexibly absorb the shocks during impacts and operation on rough terrains.

In this paper, we report on implementing LPV-based controllers for balancing our constructed autonomous bicycle in the upright position. We first present an overview of our constructed autonomous bicycle in Section 2. Next, in Section 3, we present the physical model of the bicycle. Thereafter, we present our LPV-based controller and the experimental results demonstrating the self-balancing of the bicycle at the upright position. Finally, in Section 5, we conclude the paper with some final remarks and future research directions.

### 2. OVERVIEW OF THE AUTONOMOUS BICYCLE

In this section, we present an overview of our constructed autonomous bicycle, which is depicted in Figure 1 with its dimensions and mass measurements summarized in Table 1. The autonomous bicycle has two main actuators. The first principal actuator of our autonomous bicycle is a linear electric actuator (MPC LAD-HS10) that provides high-speed actuation (65 mm/sec) for fast front wheel steering. The employed linear actuator uses integrated high-efficiency brushed DC motors and planetary gearboxes in line with the tube body, which makes it appropriate for installation in narrow and limited spaces. In mid-extension, the linear actuator extends from the seat to the handlebar in a way that it makes the front wheel of the bicycle to face straight forward. With a maximum stroke length of 10 inches, the linear actuator can accommodate for the desired steering angle range, i.e., from $-75^\circ$ to $75^\circ$. The second principal actuator of our autonomous bicycle, which allows for the self-propulsion and manned electric operation of the bicycle, is a 26-inch motorized wheel that replaces the standard rear wheel of the bicycle. The installed motorized wheel, which is taken from an e-bike kit, has its own control box that converts the 48V DC voltage to a 3-phase AC voltage in order to drive the permanent-magnet hub motor.

Two main sensors are used in the autonomous bicycle. The first sensing device is a MEMS 6-axis gyroscope and accelerometer (InvenSense MPU6050), which is employed to measure the roll angle and roll angular speed of the bicycle. MPU6050 sensor uses the I2C communication protocol to communicate with the microprocessor. The accelerometer, which measures the roll angle, is powered through the 5V output voltage of the employed microcontroller board. The second sensing device is a Hall effect sensor that is used for measuring the forward speed of the bicycle. The Hall effect sensor, which is mounted on the chainstay of the rear wheel, sends an interrupt signal to the microprocessor whenever one of the four installed magnets on the rear wheel becomes parallel with the sensor. The forward speed of the bicycle can then be calculated by using the traveled distance measurements provided by the Hall effect sensor and the readings of a timer.

| Table 1. A summary of dimensions and masses of the components in the autonomous bicycle. |
|---|---|---|---|
| **Mass** | Bicycle | 17.25 kg | Steering Electronics | 3.45 kg | Battery | 5.7 kg | Powered Wheel | 7.5 kg |
| **Height from the ground** | Center of gravity: 64.8 cm | Width: 3.8 cm | 74.5 cm | 77 cm | Center of gravity: 33 cm |

In order to have accurate roll angle measurements, the accelerometer needs to be placed on a level surface and centered across the vertical plane of the bicycle. For testing purposes, training wheels are installed on the rear wheel so that they do not touch the ground when the roll angle is changing from $-15^\circ$ to $15^\circ$. A rack,
which houses a 48V battery, is installed above the rear wheel locks and provides ample surface area for a control box housing the electronic components of the autonomous bicycle. Due to the battery heavy weight (5.7 kg), symmetrical mass distribution is achieved if the battery is placed along the center of the bicycle vertical plane.

Figure 1. The autonomous bicycle equipped with a linear electric actuator for automatic steering and its components.

3. DYNAMICAL MODEL OF THE BICYCLE

In this section, we present the linearized dynamical model of the bicycle. A realistic linearized dynamics of the roll angle should take into account the nonminimum phase nature of the bicycle steering. In particular, the nonminimum phase bicycle roll dynamics should reflect the intuitive fact that steering becomes ineffective for balancing the bicycle at rest or at very low speeds. Once the bicycle reaches a certain speed, the automatic steering system can be used for making the bicycle to follow a desired path.

In this paper, we will employ the linearized dynamics of front-wheel steering for single-track vehicles, presented in [2, Chapter 7]. The kinematic configuration of the bicycle and the actuator placement topology are shown in Figures 2a and 2b, respectively. The angles $\theta$, $\delta$, and $\theta_c$ denote the roll angle, the steering angle, and the slider-crank angle, respectively.

Figure 2. (a) The kinematic diagram of the bicycle; and (b) the actuator placement topology. The angles $\theta$, $\delta$, and $\theta_c$ denote the roll angle, the steering angle, and the slider-crank angle, respectively.
the angle between the slider-crank segments, respectively. The two fixed-length segments of the slider-crank in Figure 2b are given by $l_1$ and $l_2$, respectively. Finally, the actuator length at time $t$ is given by $L(t)$.

The roll dynamics are actuated through the steering angle dynamics. The steering angle, in turn, is actuated through the linear electric actuator. Therefore, we first derive the dynamics relating the steering angle to the linear actuator length. In order to do so, we note that the steering angle speed is equal to the rate of change of the slider-crank angle, i.e., $\dot{\theta}_c = \dot{\delta}(t)$. Therefore, using the law of cosines and from Figure 1a, the relationship

$$\dot{\delta}(t) = \frac{\sqrt{l_1^2 + l_2^2 - 2l_1l_2\cos(\theta_c(t))}}{l_1l_2\sin(\theta_c(t))} \dot{L}(t),$$  

holds between the linear actuator speed, i.e., $\dot{L}(t)$, and the front wheel steering angle speed, i.e., $\dot{\delta}(t)$.

Assuming zero rear-wheel steering angle and when the forward speed of the bicycle is equal to $U$, the roll angle dynamics are given by (see [2, Chapter 7] for the details of the derivation)

$$\tau_1 \dot{\theta} = -\frac{U^2}{\kappa_1} \left( \frac{\kappa_2}{U} \dot{\delta} + \delta \right),$$  

where $\tau_1, \kappa_1, \kappa_2$ are some constants that depend on the physical characteristics of the bicycle. In particular, these constants are given by

$$\tau_1 = \frac{I + mh^2}{mgh}, \quad \kappa_1 = g(a + b), \quad \kappa_2 = a,$$

where the lengths $b, a,$ and $h$ are equal to the distances from the projection of the center of mass to the front and rear axles in the ground plane, and the height of the bicycle, respectively. Moreover, the mass of the bicycle, the moment of inertia about the roll axis, and the gravitational acceleration constant are given by $m$, $I$, and $g$, respectively. Under the assumption that the speed of the bicycle is constant, the transfer function associated with the dynamics in (2) are given by

$$\frac{\hat{\theta}(s)}{\hat{\delta}(s)} = \frac{\frac{1}{\tau_1}(U\kappa_2 s + U^2)}{\tau_1 s^2 - 1},$$

where $\hat{\theta}(s)$ and $\hat{\delta}(s)$ are the Laplace transforms of $\theta(t)$ and $\delta(t)$, respectively.

From the transfer function in (4), several points are worth noting. First, since the transfer function in (4) has a pole in the right half plane, i.e., $s = 1/\tau_1$, the roll dynamics are unstable when there is no steering angle actuation. Second, the transfer function $\frac{\hat{\delta}(s)}{\hat{\theta}(s)}$ which relates the desired roll angle to the desired steering angle, has one zero in the right half plane which is the same as the unstable pole of the transfer function in (4). Hence, the transfer function $\frac{\hat{\delta}(s)}{\hat{\theta}(s)}$ is nonminimum phase. The nonminimum phase dynamics imply that the roll angle initially moves in the opposite direction of the provided reference steering angle. Karnopp$^2$ calls this behavior during the initial phase of steering “countersteering”, whereby the roll angle moves in the opposite direction to the final steering direction.

### 4. EXPERIMENTAL RESULTS

In this section, we present our switched LPV-based control scheme and implement it on the bicycle to achieve self-balancing at the upright position. Since we are using a linear electric actuator for automatic steering, we need to take into account the nonlinear dynamics given by (1), which depend on the slider-crank angle in a nonlinear manner. While we could have used linear matrix inequalities (LMIs) for designing our LPV-based controllers, as in the work by Cerone et al.$^{15}$ and Andreo et al.$^{14}$ we decided to employ gain scheduled PID controllers for the preliminary implementation on our riderless bicycle. In the gain scheduling control framework,$^{19}$ which belongs to the class of LPV-based control strategies, the design plants consist of a collection of linearizations about equilibrium points indexed by a group of measurable variables, which are called the scheduling variables. In our experiments, we used the roll angle as the scheduling variable and switched the PID control gains based
on its measured value. In particular, we used the following switched control law for steering angle reference rate to achieve bicycle self-balancing at the upright position:

\[
\dot{\delta}_{\text{ref}} = k_{P1}e_{\theta} + k_{D1}\dot{e}_{\theta} + \int k_{I1}e_{\theta}(\tau)d\tau, \quad \text{for } |\theta(t)| < 2^\circ, \\
\end{equation}

and

\[
\dot{\delta}_{\text{ref}} = k_{P2}e_{\theta} + k_{D2}\dot{e}_{\theta} + \int k_{I2}e_{\theta}(\tau)d\tau, \quad \text{for } |\theta(t)| \geq 2^\circ,
\]

where \(e_{\theta} = \theta_d - \theta\) is the roll angle tracking error. In order to achieve self-balancing at the upright position, \(\theta_d\) is set to zero.

In our preliminary experiments, we only used the linear actuator for automatic steering without using the second one, i.e., the e-wheel. In order to perform the experiments, the bike is pushed by an operator during which it will be given an initial disturbance. After the early pushing phase, the riderless bicycle will be moving on its own while the gain scheduled PID controller balances it at the upright position. The movement of the bicycle continues with a continual loss of momentum up to a point where it is caught by the operator. Figure 3 depicts the snapshot of the experiments with the peak forward speed equal to 3 m/s. Figures 4 depicts the snapshot of the experiments with the peak forward speed equal to 4 m/s. The roll angle and speed time profiles in both experiments associated with faster and slower peak forward speeds are depicted in Figures 5a and 5b, respectively.
5. CONCLUDING REMARKS

In this paper, we reported on implementing a linear parameter-varying based control scheme for achieving self-balancing of an autonomous bicycle with linear electric actuators in the upright position. The high-speed linear actuator along with the gain scheduled PID control law proved capable of balancing the bicycle across a variety of speed ranges. In our future research, we aim to use more advanced techniques such as intelligent and nonlinear control schemes for achieving self-balancing and maneuvering on non-straight paths with our constructed autonomous bicycle.

ACKNOWLEDGMENTS

This work was supported by the Office of Research and Sponsored Programs (ORSP) and the College of Engineering and Computer Science (CECS), University of Michigan-Dearborn.

REFERENCES


