MAC-layer and PHY-layer Network Coding for Two-way Relaying: Achievable Regions and Opportunistic Scheduling

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Abstract—We consider two-way relay channel with three nodes: two source nodes and one relay in the middle. We characterize the end-to-end rate regions achieved by MAC-layer network coding (MAC-XOR) and PHY-layer network coding (PHY-XOR). Noticing that MAC-XOR does not always achieve better performance than pure multi-hopping, we show a simple opportunistic scheduling to achieve the combined region. A similar opportunistic scheduling is also presented for PHY-XOR operation. It is shown that the system is stabilized for any Poisson arrivals with rate pair within their respective Shannon rate regions. Moreover, we also present simulation results based on practical cellular system models for both SISO and MIMO cases.

I. INTRODUCTION

Recently there are a lot of interests in the two-way relaying channel [3]–[20], depicted in Figure 1. This channel consists of two source nodes and one relay node in the middle, and is a basic building block in relay networks.

Being considered for extending cell coverage and/or improving signal strength for cellular systems [1], multi-hop relaying traditionally only considers orthogonal operation for the traffic from the two opposite directions. That is, to relay information for both directions, they need to be separated by time or frequency. So, in order for the two end-nodes in Figure 1 to exchange two packets, a 4-slot operation is typical, where a packet is hopped twice from node $X_0$ to $X_2$, and vice versa.

As network coding has shown promising gains for wireline networks [2], simple network coding operation has been considered for the two-way relay channel [3]–[6]. It can be applied here due to the following two reasons. First, the two data streams are using the same “route”, allowing typical network coding technique of mixing data streams to work. Secondly, wireless communication is broadcast in nature, allowing the relay node to send to both end-nodes at the same time. A simple operation, MAC-XOR, is thus presented as follows [3]–[6]. In the first slot the source node $X_0$ sends packet $pkt_1$ to the relay $X_1$, and in the second slot the source node $X_2$ sends its packet $pkt_2$ to the relay. In the third slot, the relay broadcasts the bit-wise XOR-ed packet $pkt_1 \oplus pkt_2$ to both end nodes. Finally node $X_0$ gets $pkt_2$ by XOR-ing the received packet with its old $pkt_1$; and similarly does $X_2$ in getting $pkt_1$. The third slot is depicted in Figure 2. In the paper we call this operation MAC-XOR due to the fact that the XOR operation is happening at or above the media access layer.

This simple idea has been shown to achieve interesting performance gains for various scenarios. Larsson et al [3], [4] applied to relaying in a cellular setting. Wu et al [5] considered combining unicast streams in a LAN setting. Katabi et al. [6], [7] presented an algorithm for mesh networks using network coding which showed significant gain in dense traffic. In [8], Effros et al described a tiling approach for network code design to accommodate multiple unicasts in a wireless network on a triangular lattice.

The two-way relay channel in Figure 1 can also be considered from a multi-user information theory point of view. It has been considered for a two-stage operation as follows. First, both end-nodes send to the relay by multi-access with successive cancelation as in classical multi-access channel [9]; and then the relay broadcasts to both end nodes. Reference [10] investigated the achievable rates, [11] considered the MIMO case, and [12] considered the scaling. In [13] the sum-rate was considered for both the two-stage operation and a three-stage operation, and [14] also considered both for a general setting where one end-node can hear the other end-user.

Focusing on the broadcast stage of the channel, many researchers noticed that each direction can achieve the in-
individual link-capacity if there exists a common input distribution maximizing mutual information for both links, e.g. the Gaussian channel [14]–[20]. It is achieved by classical Shannon random coding and joint typicality decoding, where the number of possible codewords is smaller as a receiver knows what the relay is trying to send to the other direction. Observing that the MAC-XOR operation only achieves the smaller link capacity (see, e.g. [16]), this poses an interesting question on whether one can achieve the Shannon broadcast region with simple operation like MAC-XOR. It turns out that one can achieve it by a PHY-XOR operation [14]–[17].

In the case of binary symmetric channel or QAM modulated Gaussian channel, the PHY-XOR works by first encoding each data packet into a channel code and then taking bit-wise XOR with the two channel codewords [16].

In this paper we focus on the three-stage operation, which is a more common practice than multiaccess with successive cancelation. We first characterize the end-to-end rate regions of both MAC-XOR and PHY-XOR, assuming the channel is known to every user. The results show that MAC-XOR is not always better than traditional multi-hopping while PHY-XOR is. Then we present simulation results based on practical cellular system models to characterize the area where network coding brings significant gain over traditional multi-hopping. Both SISO and MIMO are considered. Finally, we consider the scheduling when the packets arrive according to Poisson processes. An opportunistic scheduling algorithm is shown to stabilize the system for any bit-arrival rate pair within the Shannon rate region.

The rest of the paper is organized as follows. Section II presents the modeling details. Section III characterizes the Shannon rate regions and presents the simulation results. Section IV discusses scheduling of random arrivals, and finally Section V concludes.

II. MODELING ASSUMPTIONS AND PHY-XOR OPERATION

We consider the two-way relay channel in Figure 1, where node $X_0$ wants to send information to node $X_2$ with rate $R_0$ while node $X_2$ wants to send information to node $X_0$ with rate $R_2$. There is no direct link between the two end nodes, and all three nodes are half-duplex, i.e., a node cannot send and receive at the same time.

We assume that the two channels from the relay $X_1$ to $X_0$ and $X_2$ achieve their capacities for a common input distribution $p(x_1)$. For example, this could be Gaussian distribution for Gaussian channel or uniform binary input for binary symmetric channel. Denote the link capacity from node $i$ to $j$ as $C_{ij}$. As will be shown in Section III, link capacities $C_{ij}$ are enough for characterizing the achievable regions for traditional multihopping, MAC-XOR and PHY-XOR.

The PHY-XOR operation can be easily illustrated for the binary symmetric channel case as follows [15]–[17]. After the first two slots, the relay has received two binary messages $W_0$ and $W_2$, one from each end-node. In the third slot, instead of taking bit-wise XOR of $W_1$ and $W_2$, the relay encodes message $W_1$ to a channel code $X_{12}(W_0)$ according to the channel between $X_1$ and $X_2$. Similarly, $W_2$ is encoded into channel code $X_{10}(W_2)$ according to the channel between $X_1$ and $X_0$. The lengths of $W_0$ and $W_2$ are chosen such that both channel codewords have the same length. Now the relay broadcasts the bit-wise XOR-ed sequence $X_{12}(W_0) \oplus X_{10}(W_2)$ to both end-nodes. Upon receiving a bit sequence $Y_2$, node $X_2$ decodes as follows. It first generates $X_{10}(W_2)$ because it knows $W_2$. Then it takes bit-wise XOR between $Y_2$ and $X_{10}(W_2)$. Finally it decodes by sending the XOR-ed sequence into the decoder designed for single transmission from node $X_1$ to $X_2$ with encoder $X_{12}(\cdot)$. Node $X_0$ decodes similarly. The process is depicted in the following Figure 3, and has been shown to achieve the individual link capacity for each direction. For general channels, PHY-XOR can be defined in a more general way [17] to achieve the individual capacities simultaneously.

![PHY-XOR for a binary symmetric channel.](image)

The following lemma summarizes the performance of MAC-XOR and PHY-XOR for the broadcast stage [16], [17].

**Lemma 2.1:** Assume that there is a common input which maximizes both the mutual information from node $X_1$ to node $X_0$ and the mutual information from node $X_1$ to node $X_2$. Denote the respective link capacities as $C_{10}$ and $C_{12}$. Then in the broadcast stage MAC-XOR achieves rate pair $(\min(C_{10}, C_{12}), \min(C_{10}, C_{12}))$, while PHY-XOR achieves $(C_{10}, C_{12})$.

In the sequel, we use FSMH to denote the traditional four-slot multihopping operation.

III. ACHIEVABLE RATE REGIONS OF MAC-XOR AND PHY-XOR

In this section, we characterize the rate regions for both MAC-XOR and PHY-XOR.

A. Shannon Rate Regions

In this subsection, we assume that the link capacities are known to each node, and $C_{10}$ and $C_{12}$ are achieved by the same input distribution. We consider rate pairs $(R_0, R_2)$ that can be achieved by MAC-XOR and PHY-XOR.

For comparison, we first consider the rate region achieved by FSMH. Its achievable rate pairs are described by the
following constraints by definition:
\[ \mathcal{C}_{FSMH} := \{(R_0, R_2) : R_0 \leq \lambda_1 C_{01}, R_0 \leq \lambda_2 C_{12},
R_2 \leq \lambda_3 C_{21}, R_2 \leq \lambda_4 C_{10}, \sum \lambda_i = 1\}, \]
where \( \lambda_i \) are the time-sharing parameters.

We have the following result.

**Theorem 3.1:** The rate region \( \mathcal{C}_{FSMH} \) is the triangle formed by the origin, \((0, C_{012}, 0)\), and \((0, C_{210})\), where \( C_{012} := 1/(1/C_{01} + 1/C_{12}) \) and \( C_{210} := 1/(1/C_{21} + 1/C_{10}) \) are the two maximum one-way rates achieved by multi-hopping; see Fig. 4.

**Proof:** It is easy to verify that \((0, C_{012}, 0)\) is achieved by setting \( \lambda_1 = C_{012}/C_{01}, \lambda_2 = C_{012}/C_{12}, \lambda_3 = \lambda_4 = 0 \). And \((0, C_{210})\) is achieved by setting \( \lambda_3 = C_{210}/C_{21}, \lambda_4 = C_{210}/C_{10}, \lambda_1 = \lambda_2 = 0 \). Since the region is convex as it is described by linear constraints, the triangle region is achievable.

Now we show the triangle is also an outer bound for FSMH. Considering the four inequalities and dividing each one by its corresponding link capacity, we have
\[ \frac{R_0}{C_{01}} + \frac{R_0}{C_{12}} + \frac{R_2}{C_{21}} + \frac{R_2}{C_{10}} \leq \sum \lambda_i = 1. \]
Rewriting, it is \( \frac{R_0}{C_{01}} + \frac{R_2}{C_{21}} \leq 1 \), which is the region below the line connecting the two points \((0, C_{012})\) and \((0, C_{210})\).

For MAC-XOR, the rate region is described by the following constraints by definition:
\[ \mathcal{C}_{MAC-XOR} := \{(R_0, R_2) : R_0 \leq \lambda_1 C_{01}, R_2 \leq \lambda_2 C_{12},
R_0 \leq \lambda_3 \min(C_{12}, C_{10}), \sum \lambda_i = 1\}. \]
We have the following result on its achievable region.

**Theorem 3.2:** If \( C_{12} \leq C_{10} \), then the rate region \( \mathcal{C}_{MAC-XOR} \) is the quadrilateral formed by the origin, \((0, C_{012}, 0)\), \((1/(C_{01} + 1/C_{10})^{-1}, 1/(C_{01} + 1/C_{10})^{-1})\), and \((0, 1/(C_{12} + 1/C_{21})^{-1})\); see Fig. 4. If \( C_{12} > C_{10} \), then the rate region \( \mathcal{C}_{MAC-XOR} \) is the quadrilateral formed by the origin, \((1/(C_{01} + 1/C_{10})^{-1}, 0)\), \((1/(C_{210} + 1/C_{10})^{-1}, 1/(C_{210} + 1/C_{10})^{-1})\) and \((0, C_{210})\).

**Proof:** If \( C_{12} \leq C_{10} \), then the region becomes
\[ \mathcal{C}_{MAC-XOR} := \{(R_0, R_2) : R_0 \leq \lambda_1 C_{01}, R_2 \leq \lambda_2 C_{12},
R_0 \leq \lambda_3 C_{12}, \sum \lambda_i = 1\}. \]

It is easy to verify that \((0, C_{012}, 0)\) is achieved by setting \( \lambda_1 = C_{012}/C_{01}, \lambda_3 = C_{012}/C_{12} \) and \( \lambda_2 = 0 \).

Now we show the quadrilateral is also an outer bound for MAC-XOR. Considering the four inequalities and dividing each one by its corresponding link capacity, we have
\[ \frac{R_0}{C_{01}} + \frac{R_0}{C_{12}} + \frac{R_2}{C_{21}} + \frac{R_2}{C_{10}} \leq \sum \lambda_i = 1. \]
The first one corresponds to the region below the line connecting \((0, C_{012})\) and \((1/(C_{01} + 1/C_{10})^{-1}, 1/(C_{01} + 1/C_{10})^{-1})\), and the second one corresponds to the region below the line connecting \((0, 1/(C_{12} + 1/C_{21})^{-1})\) and \((1/(C_{012} + 1/C_{10})^{-1}, 1/(C_{012} + 1/C_{10})^{-1})\).

The proof for the case when \( C_{12} > C_{10} \) is similar.

For PHY-XOR, the rate region is described by the following constraints:
\[ \mathcal{C}_{PHY-XOR} := \{(R_0, R_2) : R_0 \leq \lambda_1 C_{01}, R_2 \leq \lambda_2 C_{12},
R_0 \leq \lambda_3 C_{12}, \sum \lambda_i = 1\}. \]
We have the following result on its achievable region.

**Theorem 3.3:** The rate region \( \mathcal{C}_{PHY-XOR} \) is the quadrilateral formed by the origin, \((0, C_{012}, 0)\), \((1/(C_{01} + 1/C_{10})^{-1}, 1/(C_{01} + 1/C_{10})^{-1})\), and \((0, C_{210})\); see Fig. 4.

**Proof:** It is easy to verify that \((0, C_{012}, 0)\) is achieved by setting \( \lambda_1 = C_{012}/C_{01}, \lambda_3 = C_{012}/C_{12} \) and \( \lambda_2 = 0 \). \((0, C_{210})\) is achieved similarly.

Since the region is convex as it is described by linear constraints, the quadrilateral region is achievable.

We have the following result on its achievable region.

Now we show the quadrilateral is also an outer bound for PHY-XOR. Considering the four inequalities and dividing each one by its corresponding link capacity, we have
\[ \frac{R_0}{C_{01}} + \frac{R_0}{C_{12}} + \frac{R_2}{C_{21}} + \frac{R_2}{C_{10}} \leq \sum \lambda_i = 1. \]
The first one corresponds to the region below the line connecting \((0, C_{012})\) and \((1/(C_{01} + 1/C_{10})^{-1}, 1/(C_{01} + 1/C_{10})^{-1})\), and the second one corresponds to the region below the line connecting \((0, C_{210})\) and \((1/(C_{01} + 1/C_{10})^{-1}, 1/(C_{01} + 1/C_{10})^{-1})\). The following is an immediate observation based on the above theorems.
Remark 3.4: MAC-XOR operation is not always better than FSMH. Specifically, MAC-XOR is worse than FSMH when either $C_{12} < C_{10}$ and $R_2/R_0 > 1$, or $C_{12} > C_{10}$ and $R_2/R_0 < 1$. Time-sharing between MAC-XOR and FSMH achieves a larger region, the convex hull of $C_{FSMH} \cup C_{MAC-XOR}$. Whereas the PHY-XOR region is always the best among the three.

B. Simulations for a Cellular Fading Environment

In this subsection we study the performance gain achieved by using MAC-XOR and PHY-XOR in a cellular environment. We consider one cell with a relay station 800 meters away from the base station. We want to know in which area network coding attains gain as compared to traditional multihopping. Based on the analysis we know that network coding may not achieve significant gain.

Due to the fact that in practice traffic is always of certain pattern, we impose the requirement that $R_2 = 0.75 R_0$, i.e., the uplink traffic is smaller than the downlink one. The channel model is Erceg B model [22]. In the simulation, a location is randomly picked in the cell and a channel realization is generated. For this realization, we calculate the sum-rate achieved by FSMH, MAC-XOR and PHY-XOR. If the gain is larger than 10%, then the location is marked. Both SISO case and MIMO case are considered. In MIMO case, we assume that the relay station just uses identity input covariance matrix.

Figure 5 is the performance when every station has one antenna, whilst Figure 6 is the performance when the base station has four antennas, the relay station has four and the end user has two. Based on the figures, one notices that 1) network coding brings gain in a big area, and 2) the area where network coding brings gain is even larger under MIMO.

IV. STABILIZE POISSON ARRIVALS BY OPPORTUNISTIC SCHEDULING

We have characterized the rate regions for the three operations. In this section we consider the following question. If data packets arrive at the two end nodes according to random process, how should the packet be scheduled? And what is the stable region for the rate pairs where the two queues at the two end nodes do not build up to infinity?

To answer the above questions, we model the channel as follows. We assume that there are two infinite queues at the two end nodes, one for each. A packet arrives at the queue at node $X_0$ according to Poisson process with rate $R_{0}'$, while a packet arrives at the queue at node $X_2$ according to Poisson process with rate $R_{2}'$. We also assume that all the packets have same length $L$.

For MAC-XOR operation, consider the following opportunistic scheduling.

ONC(MAC) Scheduling Algorithm : Upon finishing the previous transmission, packets are transmitted in the next transmission according to the following policy:

1) If neither queue is empty, choose one packet from each queue and send them over the relay by the MAC-XOR scheme.
2) If one queue is empty, choose one packet from the other queue and send it over the relay by the multihopping scheme.

Note that the attractive feature of this algorithm is that it does not require any arrival rate information of the two queues.
to achieve the system stability as stated in the following theorem:

**Theorem 4.1:** The ONC(MAC) scheduling algorithm stabilizes the two-way relaying system for any Poisson arrivals as long as the (bit-arrival) rate pair \((R_0,L,R_2,L)\) is within the convex hull of \(c_{FSMH} \cup c_{MAC-XOR}\).

**Proof:** Denote \(R_0 := R_0^1L\) and \(R_2 := R_2^2L\). Without loss of generality, assume each packet has unit length. Consider the time right after the \(n\)-th transmission. The queue length vector is denoted by \(Q(n) = [q_0(n), q_2(n)]\) in which \(q_0(n)\) and \(q_2(n)\) represent the two queue lengths at station \(X_0\) and \(X_2\), respectively. It is obvious that \(Q(n)\) forms a non-reducible Markov chain. Note that the number of packets arriving at \(X_0\) during a transmission time slot \(\Delta t\) is a Poisson process with parameter \(R_0\Delta t\), while the number of packets arriving at \(X_2\) is also a Poisson process with parameter \(R_2\Delta t\).

In order to show the stability of the Markov process \(Q(n)\), we define the Lyapunov function as follows:

\[
V(n) := \frac{C_{nc}}{C_{012} - C_{nc}} q_0^2(n) + \frac{C_{nc}}{C_{210} - C_{nc}} q_2^2(n) + 2q_0(n)q_2(n),
\]

where \(C_{nc} := (1/C_{21} + 1/C_{01} + 1/min(C_{10}, C_{12}))^{-1}\). If both \(q_0(n) > 0\) and \(q_2(n) > 0\), then \(q_0(n + 1) = q_0(n) - 1 + \delta_0(n)\) and \(q_2(n + 1) = q_2(n) - 1 + \delta_2(n)\) where \(\delta_0(n)\) and \(\delta_2(n)\) are a Poisson random variable with parameter \(R_0/C_{nc}\) and \(R_2/C_{nc}\) respectively. It follows that

\[
E(V(n + 1)|Q(n)) = V(n) + 2 \frac{C_{012}}{C_{nc}} \left( \frac{R_0}{C_{012}} - 1 \right) + 2 \frac{C_{210}}{C_{nc}} \left( \frac{R_2}{C_{210}} - 1 \right) q_0(n) \]

\[
+ 2 \frac{C_{012}}{C_{nc}} \left( \frac{R_0}{C_{012}} - 1 \right) + 2 \frac{C_{210}}{C_{nc}} \left( \frac{R_2}{C_{210}} - 1 \right) q_2(n) + \mathcal{E}_1(n)
\]

where \(\mathcal{E}_1(n)\) is a constant depending on \(R_0, R_2, C_{nc}, C_{012}, C_{210}\).

If \(q_0(n) > 0\) and \(q_2(n) = 0\), then \(q_0(n + 1) = q_0(n) - 1 + \delta_0(n)\), where \(\delta_0(n)\) is a Poisson random variable with parameter \(R_0/C_{012}\). Similarly, \(q_2(n + 1) = \delta_2(n)\) with \(\delta_2(n)\) being Poisson\((R_2/C_{210})\). Thus we have

\[
E(V(n + 1)|Q(n)) = V(n) + 2 \left( \frac{C_{012}}{C_{nc}} \left( \frac{R_0}{C_{012}} - 1 \right) + 2 \frac{C_{210}}{C_{nc}} \left( \frac{R_2}{C_{210}} - 1 \right) \right) q_0(n) + \mathcal{E}_2(n)
\]

where \(\mathcal{E}_2(n)\) is a constant depending on \(R_0, R_2, C_{nc}, C_{012}, C_{210}\). On the other hand, if \(q_0(n) = 0\) and \(q_2(n) > 0\) we have

\[
E(V(n + 1)|Q(n)) = V(n) + 2 \left( \frac{C_{012}}{C_{nc}} \left( \frac{R_0}{C_{012}} - 1 \right) + 2 \frac{C_{210}}{C_{nc}} \left( \frac{R_2}{C_{210}} - 1 \right) \right) q_2(n) + \mathcal{E}_3(n)
\]

where \(\mathcal{E}_3(n)\) is a constant depending on \(R_0, R_2, C_{nc}, C_{012}, C_{210}\).

Since \((R_0,R_2)\) is below the line connecting \((0,C_{210})\) and \((C_{nc},C_{nc})\) and the line connecting \((C_{012},0)\) and \((C_{nc},C_{nc})\), the following inequalities are obvious:

\[
\frac{C_{nc}}{C_{012} - C_{nc}} \left( \frac{R_0}{C_{012}} - 1 \right) + \frac{R_2}{C_{210}} < 0
\]

\[
\frac{C_{nc}}{C_{210} - C_{nc}} \left( \frac{R_2}{C_{210}} - 1 \right) + \frac{R_0}{C_{012}} < 0
\]

We therefore have \(E(V(n + 1)|Q(n)) \leq V(n) - 1\) whenever \(q_0(n)\) or \(q_2(n)\) is large. By Foster-Lyapunov criterion \([21]\), \(Q(n)\) is stable.

Now consider the scheduling for PHY-XOR. For simplicity we assume that there exists a positive number \(M > 0\) such that \((M/C_{10}, M/C_{12})\) becomes an integer pair \((m_0,m_2)\). Consider the following opportunistic scheduling.

**ONC(PHY) Scheduling Algorithm:** Upon finishing the previous transmission, packets are transmitted in the next transmission according to the following policy:

1. If the queue at node \(X_0\) has no less than \(m_0\) packets and the queue at node \(X_2\) has no less than \(m_2\) packets, then send \(m_0\) packets from \(X_0\) and \(m_2\) packets from \(X_2\) over the relay by PHY-XOR.
2. If \(X_0\) has less than \(m_0\) packets, then send \(U\) (or the full queue if its size is less than \(U\)) packets from \(X_2\) to \(X_0\) by multi-hopping, where \(U\) is a positive integer specified in the following theorem.
3. If \(X_2\) has less than \(m_2\) packets, then send \(U\) (or the full queue if its size is less than \(U\)) packets from \(X_0\) to \(X_2\) by multi-hopping.

The following theorem shows the stability region of the rate pairs:

**Theorem 4.2:** The ONC(PHY) scheduling algorithm stabilizes the two-way relaying system for any Poisson arrivals

\[\text{if there is no such M existing, one only needs to find a pair of (m_0, m_2) large enough with m_0/m_2 very close to C_{12}/C_{10}.}\]
as long as the (bit-arrival) rate pair \((R'_0L, R'_2L)\) is within 
\(\mathcal{C}_{PHY-XOR}\), and
\[
\frac{C_{nc0}}{C_{210} - C_{nc2}} \left( \frac{R'_2L}{C_{210}} - 1 \right) + \frac{R'_4L}{C_{210}} + m_0 < 0, \\
\frac{C_{nc2}}{C_{012} - C_{nc0}} \left( \frac{R'_4L}{C_{012}} - 1 \right) + \frac{R'_L}{C_{012}} + m_2 < 0,
\]
where \((C_{nc1}, C_{nc2}) := \frac{1}{(1/C_{010})+1/(C_{210}C_{12})+1/(C_{012}C_{12})}\).

Proof: Denote \(R_0 := R'_0L\) and \(R_2 := R'_2L\). Without loss of generality, assume each packet has unit length.

First notice that, since the region is enclosed by the two straight lines specified in Theorem 3.3, the number \(U\) specified in the above theorem exists.

Now consider the time right after the \(n\)-th transmission. The queue length vector is denoted by \(Q(n) = [q_0(n), q_2(n)]\) in which \(q_0(n)\) and \(q_2(n)\) represent the two queue lengths at station \(X_0\) and \(X_2\), respectively. It is obvious that \(Q(n)\) forms a non-reducible Markov chain. Note that the number of packets arriving at \(X_0\) during a transmission time slot \(\Delta t\) is a Poisson process with parameter \(R_0\Delta t\), while the number of packets arriving at \(X_2\) is also a Poisson process with parameter \(R_2\Delta t\).

In order to show the stability of the Markov process \(Q(n)\), we define the Lyapunov function as follows:
\[
V(n) := \frac{C_{nc0}q_0^2(n)}{C_{012} - C_{nc0}} + \frac{C_{nc2}q_2^2(n)}{C_{210} - C_{nc2}} + 2q_0(n)q_2(n).
\]

If step 1) is satisfied, then \(q_0(n+1) = q_0(n) - m_0 + \delta_0(n)\) and \(q_2(n+1) = q_2(n) - m_2 + \delta_2(n)\) where \(\delta_0(n)\) and \(\delta_2(n)\) are a Poisson random variable with parameter \(R_0m_0\) and \(R_2m_2\) respectively. It follows that
\[
E(V(n+1)|Q(n)) = V(n) + c_1 \left[ \frac{C_{nc0}}{C_{010} - C_{nc0}} R_0 + R_2 \left( \frac{C_{nc0}C_{nc2}}{C_{010} - C_{nc0}} + C_{nc2} \right) \right] q_0(n) \\
+ c_1 \left[ \frac{C_{nc2}}{C_{210} - C_{nc2}} R_2 + R_0 \left( \frac{C_{nc0}C_{nc2}}{C_{210} - C_{nc2}} + C_{nc0} \right) \right] q_2(n) + c_1^2
\]
where \(c_1 = 2M(1/(C_{010}) + 1/(C_{210}C_{12}) + 1/(C_{012}C_{12}))\), and \(c_1\) is a constant depending on \(R_0, R_2, C_{nc0}, C_{nc2}, C_{012}, C_{210}\).

If step 2) is satisfied, then \(q_2(n+1) = (q_2(n) - U)^+ + \delta_2(n)\) where \(\delta_2(n)\) is a Poisson random variable with parameter \(R_2U\). Similarly, \(q_0(n+1) < m_0 + \delta_0(n)\) with \(\delta_0(n)\) being Poisson \(R_0m_0\). Thus we have for \(q_2(n) > U\),
\[
E(V(n+1)|Q(n)) \leq V(n) + U \left[ \frac{C_{nc2}}{C_{210} - C_{nc2}} \left( \frac{R_2}{C_{210}} - 1 \right) + \frac{R_0}{C_{210}} + m_2 \right] q_2(n) + c_2
\]
where \(c_2\) is a constant depending on \(R_0, R_2, C_{nc0}, C_{nc2}, C_{012}, C_{210}\).

Similarly, if step 3) is satisfied, we have for \(q_0(n) > m_0\),
\[
E(V(n+1)|Q(n)) \leq V(n) + U \left[ \frac{C_{nc0}}{C_{012} - C_{nc0}} \left( \frac{R_0}{C_{012}} - 1 \right) + \frac{R_2}{C_{012}} + m_0 \right] q_0(n) + c_3
\]
where \(c_3\) is a constant depending on \(R_0, R_2, C_{nc0}, C_{nc2}, C_{012}, C_{210}\).

By choosing \(m_0, m_2\) and \(U\) as specified in the statement of the theorem, we know that \(E(V(n+1)|Q(n)) \leq V(n) - 1\) when \(q_0(n)\) or \(q_2(n)\) is large. By Foster-Lyapunov criterion [21], \(Q(n)\) is stable.

V. CONCLUDING REMARKS

In this paper we considered the three-node relay channel, which is a basic structure in wireless relay networks. The rate regions of the MAC-XOR and PHY-XOR operations for the three-node two-way relay channel are characterized. Simple scheduling algorithms are also presented to stabilize the system for any Poisson arrivals with bit-rate pair within their respective Shannon regions. It is of interest to see how the algorithms influence complex relay networks. Furthermore, it is also of interest to see whether similar simple operations exist for more general topology.

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