Math 216 Introduction to Differential Equations Winter 2018

Syllabus

3 Credit Hours

Course Meeting Times: **MTR 12 – 12:50 in CB 2048**

Format: **Recitation / Classroom Based**

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Office Hours: M 1:00 - 2:00 and TR 1:00 - 2:00 and 5:00 - 6:00. Also by appointment.

My office hours are those times I will usually be in my office. However, occasionally I have to attend a meeting during one of my regularly scheduled office hours. In this case I will leave a note on my door indicating I am unavailable. In particular, if you know in advance that you are going to come see me at a particular time, it might not be a bad idea to tell me in class just in case one of those meetings arises. Please feel free to come by to see me at times other than my office hours. I will be happy to see you.

Course Description (from Catalog):

Solutions and applications of differential equations of the first and second order, linear equations with constant coefficients, solutions by means of power series, Laplace transforms, and numerical methods for solution of differential equations

My Version of the Course Description:

A differential equation is an equation where the unknown is a function, say $x = f(t)$, and the equation involves derivatives of the unknown function. For example, find all functions $x = f(t)$ such that $\frac{dx}{dt} = -2x + 1$. The solution turns out to be $x = ½ + ce^{-2t}$ where c can be any constant. There are a number of different aspects to differential equations.

1. Modeling situations in the real world with differential equations.
2. "Solving" differential equations, either by hand, symbolically using the computer, or numerically using the computer.
3. Interpreting the solutions, i.e. drawing conclusions about what a mathematical model would predict about the real world.

We will look at each of these parts of the subject.

Most books approach the subject by breaking up differential equations into different classes and describing the solution techniques for each class in turn. We shall do pretty much the same, and the course is divided into the following seven parts.

1. Differential equations of the form $dx/dt = f(t)$.
2. First order differential equations.
3. Second order linear differential equations, especially those with constant coefficients.
4. Laplace transforms.
5. Systems of differential equations.
6. Using power series to solve differential equations.
7. Numerical solution of differential equations.

Mathematics Program Goals:

1. Increase students' command of problem-solving tools and facility in using problem-solving strategies, through classroom exposure and through experience with problems within and outside mathematics.
2. Increase students' ability to communicate and work cooperatively.

3. Increase students' ability to use technology and to learn from the use of technology, including improving their ability to make calculations and appropriate decisions about the type of calculations to make.
4. Increase students' knowledge of the history and nature of mathematics. Provide students with an understanding of how mathematics is done and learned so that students become self-reliant learners and effective users of mathematics.

Math 216 Learning Goals:

After completing this course students should be able to do the following when appropriate.

1. Model situations in the real world with differential equations
2. Solve a variety of differential equations by hand
3. Use mathematical software to solve a variety of differential equations
4. Use the solution of differential equations to answer questions about the real world situation the equations model

Texts

Fundamentals of Differential Equations, by Nagle, Saff, and Snider, published by Addison-Wesley. In the schedule below I have put in the appropriate sections and some suggested problems in the 6th edition (2004), 7th edition (2008) and the 8th edition (2011), so if you get any of these editions you should be able to follow along with the text ok. In the schedule below the 6th edition is denoted by N6, the 7th edition by N7 and the 8th edition by N8. I suggest you check out Amazon.com or other on-line sources for a cheaper price on the text. It looks like Amazon has a copies of the 6th edition for $6.00.

Student Solutions Manual for the preceding text.

Alternative Text:

Charles Bergeron, Jiri Lebl and other open-source creators, *Differential Equations Including Linear Algebra Topics and Computer-Aided Problem-Solving Using Maxima or SageMath*, 2016. To get to it, follow the link https://sites.google.com/view/differentialequations/ This book is denoted by B in the schedule below where the appropriate sections are listed and some suggested problems from this book are included.

I like Nagle, Saff and Snider a little better than Bergeron and Lebl. However, I think Bergeron and Lebl have most of the essential material.

Website:

http://www-personal.umd.umich.edu/~fmassey/math216/. This contains copies of this course outline, the assignments, exams that I gave in this course in the past and some notes that contain material from the lectures and supplementary information. The notes are abbreviated by *Notes* in the schedule below. Some of them are mainly concerned with using *Mathematica* and MATLAB to do some of the calculations that arise in the course. Most of the notes are written using the Mathematics software *Mathematica*, which you are familiar with if you took any of the courses Mathematics 115, 116 or 205 here at the University of Michigan – Dearborn. To read them you either need to use a computer on which Mathematica has been installed (many of the computers on campus have Mathematica on them) or you can use the "Mathematica Player" software that can be downloaded for free from www.wolfram.com/products/player/. This software allows you to read Mathematica files, but does not allow you to execute the Mathematica operations in the file. See me if you have trouble accessing any of the items in the website.

Assignment and Grading Distribution:

4 Midterm Exams (100 points each)	400
4 Assignments (15 points each)	60
Final Exam (100 points)	100
Total	560

The assignments can be found on CANVAS and at http://www-personal.umd.umich.edu/~fmassey/math216/Assignments/.

The dates of the exams are on the schedule below. All exams are closed book, but a formula sheet will be provided. You may find that your calculator can do some of the problems on the exams. If this is so, you still need to show how to do the problem by hand, even if you use a calculator to check your work.

In the schedule below are some suggested problems for you to work on. Some of these problems are representative of what will be on the exams, while others are simply to help you fix the concepts in your mind or prepare you to do other problems. Work as many problems as time permits and ask for help (in class or out) if you can't do them.

A copy of the formula sheet is at http://www-personal.umd.umich.edu/~fmassey/math216/Exams/Formulas.doc. No make-up exams unless you are quite sick.

Grading Scale:

On each exam and the assignments I will look at the distribution of scores and decide what scores constitute the lowest A-, B-, C-, D-. The lowest A- on each of these items will be added up and the same for B-, C-, D-. The lowest A, B+, B, C+, D+, D will be obtained by interpolation. For example, the lowest B is 1/3 of the way between the lowest B- and the lowest A-, etc. All your points will be added up and compared with the lowest scores necessary for each grade. For example, if your total points falls between the lowest B+ and the lowest A- you would get a B+ in the course.

This information is in the file YourGrade which is located in the course website at http://www-personal.umd.umich.edu/~fmassey/math216/. After each exam and assignment is graded this information will be updated and you should be able to see how you stand. You can find out what scores I have recorded for you by going to CANVAS, selecting Math 216 and clicking on Grades on the left. If possible, check your grades after each exam and assignment to see that they were entered correctly.

Withdrawal: Monday, March 19 is the last day to withdraw from the course.

Tentative Course Outline:

I will try to have the exams on the dates indicated below. However, the dates that we cover the various topics in the lecture may be a little bit different from what is indicated below.

*N*6 = Nagle, Saff, and Snider, 6th edition
*N*7 = Nagle, Saff, and Snider, 7th edition
*N*8 = Nagle, Saff, and Snider, 8th edition
N = Nagle, Saff, and Snider, 6th, 7th or 8th edition
B = Bergeron
Notes = Notes in the course website online

Dates	Section(s)	Topics and Suggested Problems
1/8, 9	*B*: §3.1 *Notes*: §1.1 – 1.4	1. Differential equations of the form $dx/dt = f(t)$. Indefinite integrals and initial value problems. Solution by hand and using mathematical software. Definite integrals, solution of initial value problems using definite integrals, special functions defined by definite integrals, solution of initial value problems using special functions. Exam 1, F17 #1 Exam 1, F15 #1 *Notes*: §1.1 #1 *Notes*: §1.2 #1, 2, 3 *Notes*: §1.3 #1 *Notes*: §1.4 #1, 2

		B: §3.1 #3, 4, 10, 104
1/11	*Assignment* 1	MATLAB: Introduction - Computations, variables, vectors, plotting, symbolic computations, scripts
1/16-18	*N*: §2.3, 3.2 - 3.5 *B*: §1.1.3, *Notes* §2.1	2. First order differential equations. First order linear equations: method of solution, existence and uniqueness, word problems, solution using mathematical software. Exam 1, F17 #2 Exam 1, F15 #2 Exam 2, F17 #4 Exam 1, F15 #5 *Notes*: §2.1.1 #1 *Notes*: §2.1.2 #1 *N*: §2.2 #37 (an interest problem) *N*: §2.3 #17, 25 (use Mathematica to do part b) *N*: §3.2 #3 (a mixing problem), 19 (a population problem), 21 (the shrinking ball problem), 27 (a radioactive decay problem) *N*: §3.3 #8, 9 (heating/cooling problems) *N*: §3.4 #1 (a motion problem) *N*: §3.5 #1, 2 (electric circuit problems) *B*: §1.1.3 #6, 7, 16, 17, 111 §3.4 #101 – 104 §3.6 #1, 2, 4, 5, 101 – 102
1/22, 23	N: §2.1, 2.2, 3.2, 3.4 *B*: §3.1 *Notes*: §2.2	Separable equations: method of solution, existence and uniqueness, word problems, solution using mathematical software. Exam 1, F17 #4 Exam 1, F15 #4 Final Exam, F17 #1 *N*6: §2.2 #11, 13, 18, 21, 27b, d (use Mathematica to do part d) *N*7: §2.2 #11, $\frac{dx}{dt} - x^3 = x$, 18, 19, 27b, d (use Mathematica to do part d) (Solution to $\frac{dx}{dt} - x^3 = x$: $x = \pm\sqrt{Ce^{2t}/(1 - Ce^{2t})}$ where $C \geq 0$) *N*8: §2.2 #11, 13, 18, 19, 27b, d (use Mathematica to do part d) *Notes*: §2.2 #1 *N*: 3.4 #13, 15 (motion problems) *B*: §3.1 #5, 6, 7, 13, 102, 103 §3.4 #102 - 106 §3.6 #7, 8
1/23	N: §2.6 *B*: §3.4 *Notes*: §2.3	Making a change of variables in a differential equation. Exam 1, F17 #3 Exam 1, F15 #3 *N*6 and *N*7: §2.6 #9, 17, 21 *N*8: §2.6 #10, 17, 21 (Solution to #10: $\ln(y^2/x^6) - y^2/x^2 = C$) *B*: §3.5 #101 - 105
1/25	N6 and *N*7: pp. 34 – 36 N8: pp. 32 – 34	Qualitative behavior of solutions of differential equations of the form $dx/dt = f(x)$. Exam 2, F17 #3 Exam 2, F15 #2 *B*: §3.7 #101 – 103

	B: §3.7 *Notes*: §2.4	*Notes*: §2.4 #1, 2
1/29		Assignment 1 due.
1/29	Assignment 2	MATLAB - First order differential equations - symbolic functions, symbolic solution of differential equations using dsolve, numerical valued functions, numerical solution of differential equations using ode45.
1/30 - 2/5	N: §4.1, 4.2 *B*: §5.1, 5.2, 5.5 *Notes*: §3.1, 3.11	3. Second order linear differential equations. Linear, 2nd order, constant coefficient, homogenous equations when the roots of the characteristic equation are real and unequal. The solution to $ax''(t) + bx'(t) + cx(t) = 0$ is $x(t) = A_1e^{r_1t} + A_2e^{r_2t}$ where r_1 and r_2 are solutions of $ar^2 + br + c = 0$. Mass on a spring systems. Solution using mathematical software. *N*: §4.2 #3, 17 (also sketch the graph of the solution) *B*: §5.2 #101, 104, 106, 107
1/30		Review for exam.
2/1		Exam #1. See the previous exams in the website.
2/5	N: §4.2 *B*: §5.2 *Notes*: §3.2	Constant coefficient equations when the roots of the characteristic equation are equal: if $r_1 = r_2$ then $x(t) = (A + Bt)e^{r_1t}$. Exam 2, F15 #1 *N*: §4.2 #1, 19 (also sketch the graph of the solution) *B*: §5.2 #102, 108
2/6 - 12	N6: §4.1 - 4.3, 4.8, 5.6 N7 and N8: §4.1 - 4.3, 4.9, 5.7 *B*: §5.2 *Notes*: §3.3, 3.4, 3.11	Constant coefficient equations when the roots of the characteristic equation are complex: If $r_1 = a + ib$ and $r_2 = a - ib$ then $x(t) = e^{at}(A \cos bt + B \sin bt)$. Electric circuits. Mechanical and electrical vibrations. Solution using mathematical software. Exam 2, F17 #1 Exam 2, F15 #3, 5 Exam 3, F17 #1 Exam 4, F17 #1 *N*6: §4.2 #3, 15 (also sketch the graph of the solution) *N*7: §4.2 #5, 19 (also sketch the graph of the solution) *N*8: §4.2 #1, 19 (also sketch the graph of the solution) *N*: §4.3 #5, 21 (also sketch the graph of the solution) *N*6: §4.8 #1, 3, 5, 7 *N*7 and *N*8: §4.9 #1, 3, 5, 7 *B*: §5.2 #103, 105, 109, 110 §5.5 #1, 4, 101, 102,
2/12		Assignment 2 due.
2/12 - 13	N: §8.5 *N*7 and *N*8: §4.7 *B*: §3.8	Euler equations. The solution to $axy''(x) + bxy'(x) + cy(x) = 0$ is $y(x) = A_1x^{r_1} + A_2x^{r_2}$ where r_1 and r_2 are solutions of $ar(r-1) + br + c = 0$. If $r_1 = r_2$ then $y(x) = (A + B \ln x)x^{r_1}$. If $r_1 = a + ib$ and $r_2 = a - ib$ then $x(t) = x^a (A \cos(b \ln x) + B \sin(b \ln x))$. Applications to temperature problems. Exam 2, F17 #2 Exam 2, F15 #4

	Notes: §3.5, 3.11	*N*: §8.5 #1, 3, 15 *B*: §5.1 #4 - 6
2/13 - 15	N6: §6.1 *N*7 and *N*8: §4.7 *B*: §5.1 *Notes*: §3.6	Linear, 2nd order, homogenous equations: existence and uniqueness, the general solution is a superposition of two solutions that are not constant multiples of each other. *Notes*: §3.6 #1 - 5
2/15 - 19	*N*7 and *N*8: §4.7 *B*: §5.3 *Notes*: §3.7	Formula for a second solution: If u is a solution to $u'' + p(t)u' + q(t)u = 0$ then another solution is $v = u \int \frac{e^{-Q}}{u^2}$ where $Q = \int p(t)\, dt$. Exam 3, F17 #2 Exam 3, F15 #1 *B*: §5.3 #3, 6, 9, 11, 12 *Notes*: §3.7 #1
2/19 - 3/5	N6: §4.4, 4.5, 4.9, 5.6 *N*7 and N8: §4.4, 4.5, 4.10, 5.7 *B*: §5.6 *Notes*: §3.8, 3.9	Undetermined coefficients: To solve $au''(t) + bu'(t) + cu(t) = de^{rt}$, substitute $u = Ae^{rt}$ and solve for A. However, if e^{rt} is a solution to the homogenous equation, try Ate^{rt} instead. To solve $au''(t) + bu'(t) + cu(t) = e^{rt}\cos(\omega t) = \mathrm{Re}[\, e^{(r+\omega)t}\,]$ first solve without Re and then take the real part of the solution. Solution using mathematical software. Mass on a spring systems and electrical circuits. Exam 3, F17 #3 Exam 3, F15 #2 Final Exam, F17 #2 *N*: §4.5 #17, 19, 21, 25 *N*6: §4.9 #1, 3, 9, 11 *N*7 and *N*8: §4.10 #1, 3, 9, 11 *N*6: §5.6 #1, 3, 5 *N*7 and *N*8: §5.7 #1, 3, 5 *B*: §5.6 #101, 102, 106, 108, 109 §5.8 #101
2/20		Review for exam.
2/22		Exam #2. See the previous exams in the website.
3/6	N: §4.6 *N*7 and *N*8: §4.7 *B*: §5.7	Variation of parameters: A solution to $u'' + p(t)u' + q(t)u = f(t)$ is $u = u_2 \int \frac{u_1 f}{W} - u_1 \int \frac{u_2 f}{W}$ where u_1 and u_2 are solutions of the homogenous equation and W is their Wronskian. Exam 3, F17 #4 Exam 3, F15 #3 Final Exam, F17 #2 Find the general solution of the differential equation $x^2y'' + 3xy' + y = 1/x$. *N*6 and *N*7: §4.6 #1, 5 *N*8: §4.6 #2, 4 (Solutions: #2: $c_1\cos 2t + c_2\sin 2t - (1/4)(\cos 2t)\ln\lvert \sec 2t + \tan 2t \rvert$ #4: $c_1e^t + c_2te^t + te^t \ln\lvert t \rvert$

		B: §5.7 #1, 3, 101, 103
3/8 - 12		Matrices
3/13	*Assignment* 3	MATLAB – Matrices in MATLAB
3/15 - 19	N: §7.1-7.3 *B*: §8.1.1 *Notes*: §4.1	4. Laplace transforms. Basic properties. Use of mathematical software. *N*: §7.2 #1, 3, 5, 7, 9, 11, 13, 15, 19 *N*: §7.3 #9 *B*: §8.1 #5 – 8, 12, 14, 15, 101, 103, 104
3/19 - 20	N: §7.4, 7.5 *B*: §8.1.2, 8.2.1, 8.2.2 *Notes*: 4.2, 4.3	Inverse Laplace transforms and application of Laplace transforms to initial value problems. Use of mathematical software. Exam 4, F17 #2 Exam 3, F15 #4 *N*: §7.4 #1, 3, 21, 25 *N*: §7.5 #5, 7 *B*: §8.1 #9 – 11, 13, 102 §8.2 #3
3/22 - 27	N: §7.6 *B*: §8.2.3 *Notes*: §4.4	Application of Laplace transforms to problems where the inhomogeneous part is a piecewise defined functions. Final Exam, F15 #1 Final Exam, F17 #3 *N*: §7.6 #1, 3, 5, 7, 9, 15, 17, 19, 33 *B*: §8.2 #2, 6, 8, 11, 101, 102
3/27		Assignment 3 due
3/27		Review for exam.
3/29		Exam #3. See the previous exams in the website.
4/2 - 5	N6: §7.9, 5.6 N7 and N8: §7.9, 5.7 *Notes*: §5.2, 5.3	5. Systems of differential equations. Solution using Laplace transforms, applications to electric circuits, solution using mathematical software. Note: Bergeron uses a different method to solve linear systems than we do in class. Final Exam, F17 #4 Final Exam, F15 #2 *N*: §7.9 #1, 3, 5, 23 *N*6: §5.6 #11 *N*7 and *N*8: §5.7 #11 *B*: §6.3 #3, 4, 103, 104
4/3	*Assignment* 4	MATLAB – Higher Order Equations and Linear Systems in MATLAB
4/9 - 12	N: §8.1-8.3 *B*: §7.1, 7.2 *Notes*: §6.1-6.3	6. Power series solutions of differential equations. Final Exam, F15 #3 *N*: §8.1 #1, 3 *N*: §8.3 #19, 21, 23 *B*: §7.2 #1, 2, 4, 101,
4/16		Assignment 4 due

4/16		Review for exam.
4/17		Exam #4. See the previous exams in the website.
4/12 - 17	N: §1.4 *B*: §3.8 *Notes*: §7.1	Numerical solutions *B*: §3.7 #4, 102
4/17	N: §1.4, 3.6 *B*: §3.8 *Notes*: §7.1, 7.2	The error in Euler's method. *N*: §1.4 #1 Just do $x = 0.1$ and 0.2. Repeat with $h = 0.05$. Using the approximations for $h = 0.1$ and $h = 0.05$, estimate the error in the approximations for $h = 0.05$. *B*: §3.8 #4, 102
4/19		Review for final.
Wednesday, April 25, 11:30 – 2:30 p.m. Final Exam.		

University Attendance Policy

A student is expected to attend every class and laboratory for which he or she has registered. Each instructor may make known to the student his or her policy with respect to absences in the course. It is the student's responsibility to be aware of this policy. The instructor makes the final decision to excuse or not to excuse an absence. An instructor is entitled to give a failing grade (E) for excessive absences or an Unofficial Drop (UE) for a student who stops attending class at some point during the semester.

Academic Integrity

The University of Michigan-Dearborn values academic honesty and integrity. Each student has a responsibility to understand, accept, and comply with the University's standards of academic conduct as set forth by the Code of Academic Conduct (http://umdearborn.edu/697817/), as well as policies established by each college. Cheating, collusion, misconduct, fabrication, and plagiarism are considered serious offenses and violations can result in penalties up to and including expulsion from the University.

Disability Statement

The University will make reasonable accommodations for persons with documented disabilities. Students need to register with Disability Resource Services (DRS) every semester they are enrolled. DRS is located in Counseling & Support Services, 2157 UC (http://www.umd.umich.edu/cs_disability/). To be assured of having services when they are needed, students should register no later than the end of the add/drop deadline of each term. If you have a disability that necessitates an accommodation or adjustment to the academic requirements stated in this syllabus, you must register with DRS as described above and notify your professor.

Safety

All students are strongly encouraged to register in the campus Emergency Alert System, for communications during an emergency. The following link includes information on registering as well as safety and emergency procedures information: http://umdearborn.edu/emergencyalert/. Finally, all students are also encouraged to program 911 and UM-Dearborn's Public Safety phone number (313) 593-5333 into personal cell phones. In case of emergency, first dial 911 and then if the situation allows call UM-Dearborn Public Safety.