**Math/CCM 404 / 504 Dynamical Systems Fall 2016**

**Syllabus**

**3 Credit Hours**

**Course Meeting Times: TuTh 6:00 – 7:15 in 2070 CB**

**Format: Recitation / Classroom Based**

**Instructor:** Frank Massey

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**Office Hours:** M 12:30 – 1:30

TuTh 12:30 – 1:30 and 5:00 – 6:00 and after class as long as there are questions

and by appointment

My office hours are those times I will usually be in my office. However, occasionally I have to attend a meeting during one of my regularly scheduled office hours. In this case I will leave a note on my door indicating I am unavailable. In particular, if you know in advance that you are going to come see me at a particular time, it might not be a bad idea to tell me in class just in case one of those meetings arises. Please feel free to come by to see me at times other than my office hours. I will be happy to see you.

**Course Description (from Catalog):**

The aim of this course is to survey the standard types of differential equations. This includes systems of differential equations, and partial differential equations, including for each type, a discussion of the basic theory, examples of applications, and classical techniques of solutions with remarks about their numerical aspects. Also included are autonomous and periodic solutions, phase space, stability, perturbation techniques and Method of Liapunov. Students cannot receive credit for both MATH 404 and MATH 504.

**My Version of the Course Description:**

This course studies systems of differential and difference equations. There are four aspects of this subject.

1. Understanding how these equations arise in science and engineering and interpreting the mathematical solution in the real world.

2. Techniques for solving these equations when possible (primarily in the linear case).

3. Use of computer software to do the calculations involved in constructing solutions or numerical approximations to the solutions.

4. Methods for understanding the behavior of the solutions, particularly when one can't get explicit formulas for the solution.

The first half of the course concentrates on the linear case and the emphasis is on solving the equations using the eigenvalues and eigenvectors of the associated matrix. The second half of the course focuses on nonlinear equations and methods for obtaining properties of the solution when one doesn't have analytical methods for solving the equations.

**Mathematics Program Goals:**

1. Increase students’ command of problem-solving tools and facility in using problem-solving strategies, through classroom exposure and through experience with problems within and outside mathematics.

2. Increase students’ ability to communicate and work cooperatively.

3. Increase students’ ability to use technology and to learn from the use of technology, including improving their ability to make calculations and appropriate decisions about the type of calculations to make.

4. Increase students’ knowledge of the history and nature of mathematics. Provide students with an understanding of how mathematics is done and learned so that students become self-reliant learners and effective users of mathematics.

**Math/CCM 404/504 Learning Goals:** After completing this course students should be able to do the following.

1. Model systems in the real world by differential and difference equations.

2. When possible, obtain solutions for these equations in terms of familiar functions.

3. Use mathematical software to obtain solutions for these equations.

4. When possible, obtain properties of the solutions without solving the equations.

5. Use the mathematical results to answer questions about the systems in the real world modeled by the differential and difference equations.

**Text:**

*Differential Equations, Dynamical Systems, and an Introduction to Chaos* by Morris W. Hirsch, Stephen Smale and Robert L. Devaney. Published by Elsevier , Either the 2nd edition (2004, ISBN 0‑12‑349703‑5) or the 3rd edition (2013, ISBN 978-0‑12‑382010-5) is fine. The library has 3rd edition as an eBook.

**Website:**

<http://www-personal.umd.umich.edu/~fmassey/math404/>. This contains copies of this course outline, the assignments, exams that I gave in this course in the past and some *Notes*. Some of the *Notes* cover material from the lectures and some are concerned with using *Mathematica* to do some of the calculations that arise in the course. Some of the notes are written using *Mathematica*, and to read them you either need to use a computer on which Mathematica has been installed (many of the computers on campus have Mathematica on them) or you can use the "Mathematica Player" software that can be downloaded for free from [www.wolfram.com/products/player/](http://www.wolfram.com/products/player/). This software allows you to read Mathematica files, but does not allow you to execute the Mathematica operations in the file. See me if you have trouble accessing any of the items in the website.

**Math/CCM 404 Assignment and Grading Distribution :**

3 Midterm Exams (100 points each) 300

Assignments 100

Final Exam 100

Total 500

**Math/CCM 504 Assignment and Grading Distribution :**

3 Midterm Exams (100 points each) 300

Assignments 100

Project – Part 1 25

Project – Part 2 25

Final Exam 100

Total 550

You may continue submitting solutions to the problems on the assignments until you have accumulated 100 points. 100 points is the maximum that your assignments may count toward your grade. The assignments are the same for both Math/CCM 404 and 504 and can be found on CANVAS and at [www-personal.umd.umich.edu/~fmassey/math404/Assignments/](http://www-personal.umd.umich.edu/~fmassey/math404/Assignments/).

The two parts of the Graduate Project can also be found on CANVAS and at [www-personal.umd.umich.edu/~fmassey/math404/Assignments/](http://www-personal.umd.umich.edu/~fmassey/math404/Assignments/).

The dates of the exams are on the schedule below. All exams are closed book, but a formula sheet will be provided. You may use a graphing calculator.

There are several sources of problems that will help you prepare for the exams. First, there are the prior exams in the website. Second, there are the assignments. Finally, there are some problems in the text and notes that are listed in the schedule below. Work on these and ask for help (in class or out) if you can’t do them.

A copy of the formula sheet is at   
[www-personal.umd.umich.edu/~fmassey/math404/Exams/Formulas.doc](http://www-personal.umd.umich.edu/~fmassey/math404/Exams/Formulas.doc). No make-up exams unless you are sick.

**Grading Scale:**

On each exam and the assignments I will look at the distribution of scores and decide what scores constitute the lowest A-, B-, C-, D-. The lowest A- on each of these items will be added up and the same for B-, C-, D-. The lowest A, B+, B, C+, D+, D will be obtained by interpolation. For example, the lowest B is 1/3 of the way between the lowest B- and the lowest A-, etc. All your points will be added up and compared with the lowest scores necessary for each grade. For example, if your total points falls between the lowest B+ and the lowest A- you would get a B+ in the course.

This information is in the file YourGrade which is located in the course website at <http://www-personal.umd.umich.edu/~fmassey/math404/> . After each exam and assignment is graded this information will be updated and you should be able to see how you stand. You can find out what scores I have recorded for you by going to CANVAS, selecting the course and clicking on Grades on the left. Check your grades after each exam and assignment to see that they were entered correctly.

**Withdrawal:** Thursday, November 10 is the last day to withdraw from the course.

**Tentative Course Outline:**

*HSD =* Hirsch, Smale and Devaney (the text)

*Notes =* On-line Notes

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| --- | --- | --- |
| Dates | Section(s) | Topics and Suggested Problems |
| 9/8 | *HSD*: §1.1, 1.2  *Notes*: §1.1 - 1.2 | 1. **Single differential equations.** Population models, differential equations of the form *dx*/*dt* = *f*(*x*), equilibrium points, sources and sinks, the phase line, separation of variables.  Exam 1, Fall 2004 #1  Exam 1, Winter 2006 #1  p. 16 #1, 2, 10a, b  A mechanical engineer uses the equation = 1 - to describe how the temperature *T* in a heat exchanger varies with time *t*. Do the following.  i. Find the equilibrium points.  ii. Classify each equilibrium point as to whether it is a source, sink, or neither. Explain why you classify each equilibrium point the way you do.  iii. Draw the phase line.  iv. Indicate the equilibrium solutions and some typical non-equilibrium solutions on the *t*-*T* plane.  v. Suppose that at time *t* = 0 the temperature is *T* = 3. Draw a sketch of the graph of the solution *T* = *T*(*t*) showing its increasing/decresing behavior and what happens as *t* → ∞. (You should not have to solve the differential equation.) |
| 9/13 | *HSD*: §1.3, 7.2 8.5 (pp. 176-179), 12.1  *Notes*: §1.3 - 1.6 | Bifurcations, moving objects, electric circuits, epidemics, existence and uniqueness.  Ex 1, F04 #2, 3  Ex 1, W06 #2, 3  Final Exam, F04 #1  p. 16 #3 (Also classify the bifurcations as saddle-node or pitchfork.) |
| 9/15 | *HSD*: §15.1  *Notes*: §2.1, 2.2 | 2. **Single difference equations**. Population models. Difference equations of the form *xn*+1 = *f*(*xn*). The equation *xn*+1 = *rxn*. Equilibrium points, sources and sinks. Graphical construction of the solution. Cases where the solution converges monotonically to an equilibrium point, the equation  *xn*+1 = *xn*(4 ‑ *xn*). Condition for the stability of an equilibrium point *x*\*, i.e. | *f*’(*x*\*) | < 1. Cases where the solution converges to an equilibrium point in an oscillatory manner, the equation *xn*+1 = *xn*(8 - *xn*).  Ex 1, F04 #4  Ex 1, W06 #4  p. 354 #1 (just find and classify the equilibrium points), 3 (The second sentence might be reworded as follows. "Show that there are four open intervals for the parameter *k* for which the behavior of orbits of *fk* is similar.") |
| 9/20 | *HSD*: §15.2  *Notes*: §2.3 – 2.4.1 | Cases where the solution converges to a periodic solution, the equation *xn*+1 = *xn*(8 - *xn*). Bifurcations.  Ex 1, F04 #5  Ex 1, W06 #5  p. 354 #1 (find and classify the periodic points that are not equilibrium points), 2, 4 |
| 9/20 | *HSD*: §15.3  *Notes*: §2.4.2 | The family of equations *xn*+1 = λ*xn*(1 - *xn*). Cases where the solution converges monotonically to an equilibrium point, cases where the solution converges to an equilibrium point in an oscillatory manner, cases where the solution converges to a periodic solution, period doubling. The family of equations *xn*+1 = ( *xn* )3 + *μ* *xn* (problems 2b and 1c on p. 354). |
| 9/22 | *HSD*: Ch. 2, §5.1‑5.2, 6.4, 13.1  *Notes*: §3.1 | 3. **Linear Systems.** Systems of linear differential equations *dX*/*dt* = *AX* where *A* is a matrix and *X* is a vector function of *t*. Mass‑spring systems. Solution when the eigenvalues of *A* are real and distinct. The matrix exponential *etA* and expressing the solution of *dX*/*dt* = *AX* with *X*(0) = *X*o in terms of the matrix exponential *X*(*t*) = *etAX*o.  Ex 2, F04 #1c, d, 4  p. 36 #2, 5  p. 36 #6 and p. 58 #4 in the case of real and distinct eigenvalues. Also find *etA* when *A* = . |
| 9/27 | *HSD*: §3.1  *Notes*: §3.2 | Equilibrium points, sources and sinks, stability, classification using the sign of the eigenvalues of *A*. The phase plane and trajectories. |
| 9/27 | *HSD*: §3.1, 3.4  *Notes*: §3.3 | Phase portraits.  Ex 2, F04 #1e  p. 58 #2(i) and p. 138 #12(d). Also, find the solution to  = *y*  = *x*  which satisfies *x*(0) = 2 and *y*(0) = 3. Indicate the trajectory corresponding to this solution on the phase portrait.  p. 36 #3  p. 58 Do what is asked for in #2 for the system  = -2*x* + *y*  = *x* – 2*y* |
| 9/27 | *Notes*: §3.6.1 | Systems of linear difference equations *Xn*+1 = *AXn* where *A* is a matrix and *X* is a vector function of *n*.  Ex 2, F04 #1a, b  Ex 2, W06 #1  Let *A* = . Find a formula for *An* and use this to solve the difference equations  *xn*+1 = *yn*  *yn*+1 = *xn* with the initial conditions *x*o = 3 and *y*o = 2. |
| 9/29, 10/4 | *HSD*: §3.2, 3.4, 5.3, 6.2, 12.1  *Notes*: §3.4 | Solution of *dX*/*dt* = *AX* when the eigenvalues of *A* are complex. Electric circuits. Phase portraits in this case.  Ex 2, W06 #2, 4  p. 57 #1, 2iii, 3a, 4 in the case of complex eigenvalues. Also find *etA* when *A* = .  p. 138 #12(b) (express the answer in terms of real valued functions) |
| 10/4 |  | Review for exam. |
| 10/6 |  | Exam 1. |
| 10/4 | *Notes*: §3.5.2 | Solution of *Xn*+1 = *AXn* when the eigenvalues of *A* are complex.  Ex 2, F04 #2  Let *A* = . Find a formula for *An* and use this to solve the difference equations  *xn*+1 = 7*xn* + 6*yn*  *yn*+1 = -3*xn* + *yn* with the initial conditions *x*o = 2 and *y*o = 4. |
| 10/11 | *HSD*: §3.3, 5.5, 6.3  *Notes*: §3.6 | Solution of *dX*/*dt* = *AX* and *Xn*+1 = *AXn* when the eigenvalues of *A* are repeated.  Exam 2, F04 #3  Exam 2, W06 #3  p. 58 #2iv, 3b, 4 in the case of repeated eigenvalues. Also find *etA* when *A* = .  p. 138 #12(c) |
| 10/13 | *HSD*: §6.5  *Notes*: §3.7 | Solution of inhomogeneous equations *dX*/*dt* = *AX* + *f*(*t*), undetermined coefficients and variation of parameters.  Exam 3, F04 #1 |
| 10/20, 25 | *HSD*: §7.1 – 7.3, 9.1, ch 11  *Notes*: §4.1 – 4.4 | 4. **Nonlinear systems.** Systems of nonlinear differential equations *dX*/*dt* = *F*(*X*) where *X* is a vector function of *t* and *F*(*X*) is a vector function of *X*. Numerical solution using computer software. Existence and uniqueness. The phase plane and phase space. Equilibrium points, nullclines, sources, sinks and saddles. Population models.  Exam 3, F04 #3a, b, c  Exam 3, W06 #2a, b, c  Final exam, F04 #2a, b, c, 3a, b, c  Final exam, W06 #1a, b, c, 2a, b, c, f, 3a, b, c  p. 185 #4 (Just find the equilibrium points, the nullclines, the regions where the vector field points northeast, northwest, southeast, and southwest, indicate these regions on the phase portrait and any other information about the phase portrait that you can determine.)  p. 211 #1  p. 253 #1 |
| 10/27 | *HSD*: §7.4, 8.1 ‑ 8.3  *Notes*: §4.5-4.6 | Linearization about equilibrium points.  Exam 3, F04 #3d, e  Exam 3, W06 #2d, e  Final exam, W06 #3d, e, f, g  p. 184 #1 |
| 11/1 | *HSD*: ch 11  *Notes*: §5.1 | Epidemic, competing species and predator-prey models |
| 11/3, 8 | *HSD*: ch. 10, §12.1-12.3  *Notes*: §5.3-5.5 | Periodic solutions and limit cycles, Lienard's and van der Pol's equations, Bendixson's criterion for the non-existence of limit cycles, the Poincare-Bendixson theorem for the existence of limit cycles. |
| 11/8 |  | Review for exam. |
| 11/10 |  | Exam 2. |
| 11/15 | *HSD*: §9.4  *Notes*: §4.6 | Conservation Laws: quantities that are conserved, i.e. constant along trajectories, construction of the phase portrait in the case of two independent variables, solution in some special cases. Frictionless motion in one dimension governed by equations of the form *d*2*x*/*dt*2 = *f*(*x*), conservation of energy.  Final exam, F04 #2d, e, f  Final exam, W06 #1d, e, f, g  Consider an electric circuit consisting of a single loop with an inductor with inductance *L* and a capacitor with capacitance *C*. Let *q* be the charge on the top plate of the capacitor and *i* be the current in the circuit where the positive direction in the loop is toward the top plate of the capacitor. One has = *i* *L* = -  In physics it is shown that the energy stored in a capacitor is *q*2/2*C* and in an inductor is *Li*2/2, so the total stored energy is *E* = *q*2/2*C* + *Li*2/2. (a). Show that energy is conserved by solutions of the above system. (b) Show how you could have "discovered" that energy was conserved if you had not been told this. (c) Use the fact that energy is conserved to sketch the phase portrait.  Find a conservation law for the following system and use this to help draw the phase portrait.  *dx*/*dt* = - *x* – *xy*2 *dy/dt* = - *y* - *x*2*y* |
| 11/17 | *HSD*: §8.4, 8.5, 9.2 | Dissipative systems: quantities that are decreasing (or non-increasing) along trajectories, Liapunov functions, conditions for stability of an equilibrium point, dissipation of energy in nonlinear oscillations with friction governed by equations of the form *d2x/dt2 = f(x*) – *g*(*dx*/*dt*).  Final exam, W06 #2d, e, g  Final exam, F04 #3d. e  p. 212 #6 (The function *L*(*x*,*y*) that you find only has to be a Liapunov function in some disk about the origin, not the entire *xy* plane.)  Show that *L*(*x*,*y*) = *x*2 + *y*2 is a Liapunov function for the following system and use this to help draw the phase portrait.  *dx*/*dt* = - *x* – *xy*2 *dy/dt* = - *y* - *x*2*y*  Show that *L*(*x*,*y*) = -2*x*2 - *y*2 is a Liapunov function for the following system and use this to help draw the phase portrait.  *dx*/*dt* = *x*3 – *y*3 *dy/dt* = 2*xy*2 + 4*x*2*y* + 2*y*3 |
| 11/22, 29 | *HSD*: §9.4, 13.1‑13.5 | Mechanical systems, motion in two and three dimensions and motion of systems consisting of more than one object, central force motion, Hamiltonean systems, conservation of area in the phase plane.  p. 213 #8a, c, e (These should be Hamiltonean systems) |
| 11/29 |  | Review for exam. |
| 12/1 |  | Exam 3. |
| 12/6 | *HSD*: §4.1, 4.2 | How properties of the matrix *A* are reflected in properties of the solution of *dX*/*dt* = *AX*. Classes of matrices: symmetric, positive and negative-definite, orthogonal. Determining properties of *A* and the solution of *dX*/*dt* = *AX* without finding the eigenvalues of *A*. |
| 12/6 | *HSD*: ch 14, 15  *Notes*: §5.6 | Attractors, sensitivity to initial conditions, density of orbits in the attractor, the Lorenz equation, *chaotic* systems.. |
| 12/8 , 13 |  | Control Problems |
| 12/13 |  | Review for final. |
| **Tuesday, December 20, 6:30 – 9:30, Final Exam.** | | |

**University Attendance Policy:**

A student is expected to attend every class and laboratory for which he or she has registered. Each instructor may make known to the student his or her policy with respect to absences in the course. It is the student’s responsibility to be aware of this policy. The instructor makes the final decision to excuse or not to excuse an absence. An instructor is entitled to give a failing grade (E) for excessive absences or an Unofficial Drop (UE) for a student who stops attending class at some point during the semester.

**Academic Integrity Policy:**

The University of Michigan-Dearborn values academic honesty and integrity. Each student has a responsibility to understand, accept, and comply with the University’s standards of academic conduct as set forth by the Code of Academic Conduct (<http://umdearborn.edu/697817/>), as well as policies established by each college. Cheating, collusion, misconduct, fabrication, and plagiarism are considered serious offenses and violations can result in penalties up to and including expulsion from the University.

In this course, the penalty for a first violation will be a grade 0 on the related assignment. A second violation will result in a failing grade for the course. All violations will be reported to CASL and the student’s home unit.

**Disability Statement:**

The University will make reasonable accommodations for persons with documented disabilities. Students need to register with Disability Resource Services (DRS) every semester they are enrolled. DRS is located in Counseling & Support Services, 2157 UC <http://www.umd.umich.edu/cs_disability/>.  To be assured of having services when they are needed, students should register no later than the end of the add/drop deadline of each term. If you have a disability that necessitates an accommodation or adjustment to the academic requirements stated in this syllabus, you must register with DRS as described above and notify your professor.

**Safety:**

All students are encouraged to program 911 and UM-Dearborn’s University Police phone number (313) 593-5333 into personal cell phones. In case of emergency, first dial 911 and then if the situation allows call University Police.

The Emergency Alert Notification (EAN) system is the official process for notifying the campus community for emergency events. All students are strongly encouraged to register in the campus EAN, for communications during an emergency. The following link includes information on registering as well as safety and emergency procedures information: <http://umdearborn.edu/emergencyalert/>.

If you hear a fire alarm, class will be immediately suspended, and you must evacuate the building by using the nearest exit. Please proceed outdoors to the assembly area and away from the building. Do not use elevators. It is highly recommended that you do not head to your vehicle or leave campus since it is necessary to account for all persons and to ensure that first responders can access the campus.

If the class is notified of a shelter-in-place requirement for a tornado warning or severe weather warning, your instructor will suspend class and shelter the class in the lowest level of this building away from windows and doors.

If notified of an active threat (shooter) you will Run (get out), Hide (find a safe place to stay) or Fight (with anything available). Your response will be dictated by the specific circumstances of the encounter.