**Math 420 / 520 Stochastic Processes Winter 2015**

**Syllabus**

**3 Credit Hours**

**Course Meeting Times: TuTh 3:30 – 4:45 in 2063 CB**

**Format: Recitation / Classroom Based**

**Instructor:** Frank Massey

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**Office Hours:** M 2:00 – 2:50

TuTh 1:45 – 2:45 and after class as long as there are questions

and by appointment

My office hours are those times I will usually be in my office. However, occasionally I have to attend a meeting during one of my regularly scheduled office hours. In this case I will leave a note on my door indicating I am unavailable. In particular, if you know in advance that you are going to come see me at a particular time, it might not be a bad idea to tell me in class just in case one of those meetings arises. Please feel free to come by to see me at times other than my office hours. I will be happy to see you.

**Course Description (from Catalog):**

Review of distribution theory. Introduction to stochastic processes, Markov chains and Markov processes, counting, and Poisson and Gaussian processes. Applications to queuing theory.

**My Version of the Course Description:**

A stochastic process is a mathematical model of a real world system that evolves in time with an element of chance in the way the future depends on the present and past. Some important examples are waiting lines and inventories. Most of the course is devoted to Markov processes which are stochastic processes where the probabilities of future events are independent of the past once the present is known. We shall be interested in the following aspects of stochastic processes: theory, applications, computations, and use in decision making.

**Mathematics Program Goals:**

1. Increase students’ command of problem-solving tools and facility in using problem-solving strategies, through classroom exposure and through experience with problems within and outside mathematics.

2. Increase students’ ability to communicate and work cooperatively.

3. Increase students’ ability to use technology and to learn from the use of technology, including improving their ability to make calculations and appropriate decisions about the type of calculations to make.

4. Increase students’ knowledge of the history and nature of mathematics. Provide students with an understanding of how mathematics is done and learned so that students become self-reliant learners and effective users of mathematics.

**Math 420/520 Learning Goals:** After completing this course students should be able to do the following.

1. Model systems in the real world by stochastic processes.

2. When possible, obtain solutions for questions involving stochastic processes in terms of familiar functions.

3. Use mathematical software to obtain solutions for these questions and to simulate stochastic processes.

4. Use the mathematical results to answer questions about the systems in the real world modeled by stochastic processes.

**Text:**

*Essentials of Stochastic Processes*, 2nd edition, by Richard Durrett, published by Springer, 2012. ISBN 978-1-4614-3614-0 and ISBN 978-1-4614-3615-7 (eBook). This is available in our library as an eBook. If you want a hard copy then try Amazon (about $40 used when I looked)

There is a 3rd edition (ISBN 978-3319456133) published in 2016. It is was about $43 used at Amazon. I haven't had a chance to compare it with the 2nd edition and the references in the schedule below are to the 2nd edition.

**Alternative Text:**

*Introduction to Probability Models*, by Sheldon M. Ross, Harcourt – Academic Press, Any of the recent editions should be fine. (11th ed. 2014. $52 used at Amazon). (10th ed. 2010. ISBN: 978-0-12-375686-2. $43 used at Amazon) (9th edition 2007. ISBN: 978-0-12-598062-2. $10 used at Amazon)

**Website:**

[www-personal.umd.umich.edu/~fmassey/math420/](http://www-personal.umd.umich.edu/~fmassey/math420/). This contains copies of this course outline, the assignments, exams that I gave in this course in the past and some notes that contain supplementary information. The notes are abbreviated by *Notes* in the schedule below. Much of the notes are written using the Mathematics software *Mathematica*, which you are familiar with if you took any of the courses Mathematics 115, 116 or 205 here at the University of Michigan – Dearborn. To read them you either need to use a computer on which Mathematica has been installed (many of the computers on campus have Mathematica on them) or you can use the "Mathematica Player" software that can be downloaded for free from [www.wolfram.com/products/player/](http://www.wolfram.com/products/player/). This software allows you to read Mathematica files, but does not allow you to execute the Mathematica operations in the file. See me if you have trouble accessing any of the items in the website.

**Math/CCM 420 Assignment and Grading Distribution:**

3 Midterm Exams (100 points each) 300

Assignments 100

Final Exam 100

Total 500

**Math/CCM 520 Assignment and Grading Distribution :**

3 Midterm Exams (100 points each) 300

Assignments 100

Project – Part 1 25

Project – Part 2 25

Final Exam 100

Total 550

You may continue submitting solutions to the problems on the assignments until you have accumulated 100 points. 100 points is the maximum that your assignments may count toward your grade. The assignments are the same for both Math/CCM 404 and 504 and can be found on CANVAS and at [www-personal.umd.umich.edu/~fmassey/math420/Assignments/](http://www-personal.umd.umich.edu/~fmassey/math420/Assignments/).

The two parts of the Graduate Project can also be found on CANVAS and at [www-personal.umd.umich.edu/~fmassey/math420/Assignments/](http://www-personal.umd.umich.edu/~fmassey/math420/Assignments/).

The dates of the exams are on the schedule below. All exams are closed book, but a formula sheet will be provided. You may use a graphing calculator.

There are several sources of problems that will help you prepare for the exams. First, there are the prior exams in the website. Second, there are the assignments. Finally, there are some problems in the text and notes that are listed in the schedule below. Work on these and ask for help (in class or out) if you can’t do them.

A copy of the formula sheet is at   
[www-personal.umd.umich.edu/~fmassey/math420/Exams/Formulas.doc](http://www-personal.umd.umich.edu/~fmassey/math420/Exams/Formulas.doc). No make-up exams unless you are sick.

**Grading Scale:**

On each exam and the assignments I will look at the distribution of scores and decide what scores constitute the lowest A-, B-, C-, D-. The lowest A- on each of these items will be added up and the same for B-, C-, D-. The lowest A, B+, B, C+, D+, D will be obtained by interpolation. For example, the lowest B is 1/3 of the way between the lowest B- and the lowest A-, etc. All your points will be added up and compared with the lowest scores necessary for each grade. For example, if your total points falls between the lowest B+ and the lowest A- you would get a B+ in the course.

This information is in the file YourGrade which is located in the course website at <http://www-personal.umd.umich.edu/~fmassey/math420/>. After each exam and assignment is graded this information will be updated and you should be able to see how you stand. You can find out what scores I have recorded for you by going to CANVAS, selecting the course and clicking on Grades on the left. Check your grades after each exam and assignment to see that they were entered correctly.

**Withdrawal:** Monday, March 20 is the last day to withdraw from the course.

**Tentative Course Outline:**

Tentative Schedule

(I will try to have the exams on the dates indicated below. However, the dates that we cover the various topics in the lecture may be a little bit different from what is indicated below.)

(*D* = Durrrett

*R* = 9th or 10th edition of Ross

*R*9 = 9th edition of Ross

*R*10 = 10th edition of Ross)

*Notes* = Online notes

|  |  |  |
| --- | --- | --- |
| Dates | Section(s) | Topics and Suggested Problems |
| 1/10 | *D*: App 1  *R:* 1.1 - 1.3  *Notes* 1.1 | 1. Discrete Probability  1. Outcomes, sample spaces, and events. Probabilities of outcomes and events. Properties of probability: additivity, non-negativity, normalization.  Ex 1, W 11 #3a, b  Ex 1, W 15 #2c  *R:* Chapter 1 #1-4, 8, 9 |
| 1/10, 12 | *D*: App 1  *R:* 2.1, 2.2  *Notes* 1.2 | 2. Repetitions, random variables, probability mass and cumulative distribution functions. Uniform, Poisson and geometric distributions. Joint probability mass functions.  *R:* ch 2 #4, 9, 34, 38 |
| 1/12 | *D*: App 1  *R:* 1.4, 1.5, 2.5, 3.2  *R*9: 2.8, *R*10: 2.9  *Notes* 1.3 | 3. Conditional probability and independent events. Conditional probability mass functions.  Exam 1, W 15 #1, 4a, b  Exam 1, W 11 #1, 4 |
| 1/17 | *D*: App 1  *R:* 2.2  *Notes* 1.4, 1.5 | 4. Independent random variables, stochastic processes, repeated independent trials, permutations and combinations, binomial probability distributions, random walks  Ex 1, W 15 #2, 3a, b  Ex 1, W 11 #2  *R:* ch 1 #3, 5, 17, 19, 25, 27  *R:* ch 2 #11, 12, 16, 21, 23 |
| 1/19 | *D*: App 1, 2  *R:* 2.4, 2.7  *Notes* 1.6, 1.7 | 5. Sums and means (or expected values) of random variables. The law of large numbers, rewards and costs.  Ex 1, W 15 #3c, d  Ex 1, W 11 #3c  *R:* ch 2 #39, 40, 47 |
| 1/21 | *Notes* 1.8 | 6. Single period inventory problems.  Resolve the newsstand problem discussed in class if the demand for newspapers has the following distributions (Here *fk* = Pr{*D* = *k*}. In each case assume the retail price is $1.00, the wholesale price is $.60 and the salvage price is $.40.  1. *fk* = ( 3 - | 3 – *k* | ) / 9 for *k* = 1, 2, 3, 4, 5 and *fk* = 0 otherwise.  2. *fk* = 2/3*k*+1 for *k* = 0, 1, 2, …  3. *fk* = *e*-2 2*k* / *k*! for *k* = 0, 1, 2, … (I don't know an explicit formula for the cumulative distribution in this case.)  Ex 1, W 15 #4  Ex 1, W11 #5 |
| 1/23 | *D*: §1.1  *R:* §4.1  *Notes* 2.1 | 2. Markov chains  a. Basic properties: definition, transition matrices, rewards and costs.  Ex 2, W 15, #1a, b, 2a  Ex 2, W 11 #1a, b, c  *D*: §1.12 #1.4, 1.5, 1.6, 1.7, 1.29 (a), 1.30 (a), (b)  *R:* ch 4 #1, 7, 18b Also, in the context of #1, suppose we start out with 1 white ball in the first urn. Find the probabilities that there will be *i* white balls in the first urn after two steps. In the context of #7, what is the probability that it will rain the day after tomorrow? |
| 1/31 |  | Review |
| 2/2 |  | Exam 1 |
| 1/31, 2/7 | *D*: §1.2  *R:* §4.2  *Notes* 2.1, 2.2 | b. Formula for the probability of being in a state after a certain number of transitions.  Ex 2, W 11 #1d, e, f  Ex 2, W 15 #1c, d |
| 2/7 | *D*: 1.4, 1.5  *R:* 4.4  *Notes* 2.3 | c. Long run behavior and steady state probabilities, long run rewards and costs.  Ex 2, W 15 #1e, f, g  Ex 2, W 11 #1g, h  *D*: §1.12 #1.9(a), 1.18, 1.26, 1.36, 1.37, 1.43  *R:* ch 4 In the context of #1, find the steady state probabilities that there will be *i* white balls in the first urn after two steps., 25 |
| 2/9 | *Notes* 2.4 | d. Inventory problems. |
| 2/9, 14 | *D*: 1.3, 1.8  *R:* 4.6  *Notes* 2.5, 2.6 | e. Time to reach a state, probability of reaching a state by a given time, probability of reaching a state for the first time at a given time, probability of reaching (or returning to) a state at all, transient and recurrent states.  Ex 2, W 15 #2a, b  Ex 2, W 11 #2b, c  *D*: §1.12 #1.55, 1.56, 1.57, 1.58  *R:* ch 4 #14, 63 |
| 2/16 | *D*: 1.3, 1.5, 1.9  *R:* 4.6  *Notes* 2.7 | f. Number of visits to a state and its expected value, total cost (or profit) and its expected value, expected time to reach a state.  Ex 2, W 15 #2c, d  Ex 2, W 11 #2d  *D*: §1.12 #1.60, 1.61, 1.62  *R:* ch 4 #63 |
| 2/21 | *D*: 2.1, App 1  *R:* 2.3  *Notes* 3.1 | 3. Continuous Probability  1. Continuous random variables. Probability density functions and cumulative distribution functions. Uniform and exponential distributions. The memory-less property of exponential random variables. Independent random variables. Expected values of random variables.  Ex 1, W 11 #1a, b, c, d  Ex 3, W 15 #1  *D*: §2.6 #2.1  *R:* ch 2 #38 |
| 2/21, 23 | *D*: 2.1, App 1, 2  *R:* 2,4 2.5, 3.3,  *Notes* 3.2, 3.3 | 2. Several random variables. Joint and conditional density functions. Sums and other functions of random variables. The minimum of exponential random variables and the probability that one exponential random variable is less than another.  Ex 3, W 15 #1, 2  Ex 3, W 11 #1e, 2a, b  *D*: §2.6 #2.4  *R:* ch 2 #36 |
| 2/23, 3/7 | *D*: 2.2  *R:* 5.1, 5.2  *Notes* 3.4 | 3. Sums of independent exponential random variables with the same mean, the Erlang distribution. Probabilities of the number of occurrences in a given time, the Poisson distribution.  Ex 3, W 11 #2c, d  Ex 3, W 15 #3  *R:* ch 5 #1, 2, 5, 9  *Notes*: §3.4 #1 |
| 3/14 | *R:* 6.1 – 6.3  *Notes* 4.1, 4.2 | 4. Continuous time Markov processes  a. Continuous time stochastic processes, specification of stochastic processes from the distribution of times in states and the probabilities of transitions to various states especially the case where the times in states are exponential random variables, transition rates and the generator matrix, specifying a Markov process by specifying the transition rates.  Ex 3, W 15 #4a  Ex 3, W 11 #3a  *R:* ch 6  #1 (I think the sentence "*In a small colony any particular male is likely to mate with any particular female in any time interval of length h, with probability λh + o(h).*" means that if there are *n* males and *m* females then the time until some female produces an offspring is an exponential random variable with mean 1/(*nmλ*),  2, 3 (Assume that if one machine is broken and then the other one breaks then the repairman continues to work on the first machine that was broken and the time to repair it is an exponential random variable with mean 1/*μ*. After the first one is fixed, then he goes to work on the second one.),  4, 5, 7, 12a |
| 3/7 |  | Review. |
| 3/9 |  | Exam 2. |
| 3/14 | *R:* 6.4  *Notes* 4.3, 4.4 | d. The Markov property, the transition probabilities *pij*(*t*), the Chapman-Kolmogorov equation and Kolmogorov differential equations. |
| 3/14 | *R:* 6.8  *Notes* 4.5 | e. Calculation of time-dependent probabilities using the matrix exponential.  Ex 3, W 11 #3b, c, d, e  Ex 3, W 15 #4b, c, d |
| 3/16 | *R:* 6.5  *Notes* 4.6 | f. Long run behavior and steady state probabilities.  Ex 3, W 15 #4e  *R:* ch 6 #12b, 13 (assume that if a potential customer arrives and finds two customers already in the barbershop, then that person leaves without waiting), 14-19 |
| 3/16 | *Notes* 4.7 | g. Rewards and costs.  Ex 3, W 15 #3e |
| 3/21 | *R:* 8.1 – 8.3.1  *Notes* 5.1 | 5. Queues  a. Single server queues with infinite capacity  *R:* ch 8 #2, 3 |
| 3/21 | *R:* 8.9.2  *Notes* 5.2 | b. Multiple server queues with infinite capacity  *R:* ch 6 #7, 14 |
| 3/23 | *R:* 8.3.2, 8.3.3  *Notes* 5.3 | c. Queues with finite capacity |
| 3/23 | *R:* 8.4  *Notes* 5.4 | d. Queueing networks  Final Ex, W 15 #1  Final Ex, W 11 #1 |
| 3/28, 30 | *R:* 2.3.4, 2.4.3, 2.5.3, 2.7  *Notes* 6.1-6.2 | 6. Brownian motion  a. The normal distribution, variances and standard deviations, the central limit theorem.  *R:* ch 2 #53, 54c, d, 68ii  Final Ex, W 15 #2  Final Ex, W11 #2 |
| 3/30, 4/4 | *R:* 10.1, 10.3.1, 10.3.2  *Notes* 6.3 – 6.4 | b. Brownian motion as the limit of a random walk, application to the diffusion of molecules in a gas, formulas for the probability of being in a certain place at a certain time, geometric Brownian motion, Brownian motion with drift.  Final Ex, W 15 #3  Final Ex, W11 #3 |
| 4/4 |  | Review. |
| 4/6 |  | Exam 3. |
| 4/11 | *R:* 10.4  *Notes* 6.5 | c. Pricing stock options, reflected Brownian motion.  Final Ex, W 11 #4  Final Ex, W 15 #4 |
| 4/13 - 20 | *R:* 7.1 – 7.4  *Notes* ch 7 | 7. Other Markov processes where the states are continuous instead of discrete.  a. Renewal theory and machine replacement: sums of independent, identically distributed, non-negative random variables, number of occurrences in a given period of time, limit theorems, rewards and costs, machine replacement. Distribution of the number of occurrences in a given period of time and the expected number of occurrences in a given period of time. The renewal equation.  *R:* ch 5 #54, 57 |
|  |  | b. Autoregressive processes. |
| 4/20 |  | Review for final exam. |
| **Thursday, April 27, 3:00 – 6:00 p.m. Final Exam.** | | |

**University Attendance Policy:**

A student is expected to attend every class and laboratory for which he or she has registered. Each instructor may make known to the student his or her policy with respect to absences in the course. It is the student’s responsibility to be aware of this policy. The instructor makes the final decision to excuse or not to excuse an absence. An instructor is entitled to give a failing grade (E) for excessive absences or an Unofficial Drop (UE) for a student who stops attending class at some point during the semester.

**Academic Integrity Policy:**

The University of Michigan-Dearborn values academic honesty and integrity. Each student has a responsibility to understand, accept, and comply with the University’s standards of academic conduct as set forth by the Code of Academic Conduct (<http://umdearborn.edu/697817/>), as well as policies established by each college. Cheating, collusion, misconduct, fabrication, and plagiarism are considered serious offenses and violations can result in penalties up to and including expulsion from the University.

In this course, the penalty for a first violation will be a grade 0 on the related assignment. A second violation will result in a failing grade for the course. All violations will be reported to CASL and the student’s home unit.

**Disability Statement:**

The University will make reasonable accommodations for persons with documented disabilities. Students need to register with Disability Resource Services (DRS) every semester they are enrolled. DRS is located in Counseling & Support Services, 2157 UC <http://www.umd.umich.edu/cs_disability/>.  To be assured of having services when they are needed, students should register no later than the end of the add/drop deadline of each term. If you have a disability that necessitates an accommodation or adjustment to the academic requirements stated in this syllabus, you must register with DRS as described above and notify your professor.

**Safety:**

All students are encouraged to program 911 and UM-Dearborn’s University Police phone number (313) 593-5333 into personal cell phones. In case of emergency, first dial 911 and then if the situation allows call University Police.

The Emergency Alert Notification (EAN) system is the official process for notifying the campus community for emergency events. All students are strongly encouraged to register in the campus EAN, for communications during an emergency. The following link includes information on registering as well as safety and emergency procedures information:<http://umdearborn.edu/emergencyalert/>.

If youhear a fire alarm, class will be immediately suspended, and you mustevacuate the building by using the nearest exit. Please proceed outdoors to the assembly area and away from the building. Do not use elevators. It is highly recommended that you do not head to your vehicle or leave campus since it is necessary to account for all persons and to ensure that first responders can access the campus.

If the class is notified of a shelter-in-place requirement for a tornado warning or severe weather warning, your instructor will suspend class and shelter the class in the lowest level of this building away from windows and doors.

If notified of an active threat (shooter) you will Run (get out), Hide (find a safe place to stay) or Fight (with anything available). Your response will be dictated by the specific circumstances of the encounter.