Velocity

Motion Function x(t)

The *motion* of an object in one dimension is described by the "motion function" x(t):

x(t) ≡ position x of the object as a function of time t .

The *velocity* of the object is defined as the rate of change of the position with respect to time:

$v \equiv dx/dt$.

In this experiment, you will focus on the special case of motion at *constant velocity*. You will use a *motion sensor* to measure the motion function x(t) and the velocity v.

Spacetime and Worldline

Space x and time t are the fundamental entities that characterize the phenomenon of motion. The coordinate system with x on the vertical axis and t on the horizontal axis is called a *Spacetime Diagram*. A graph of the motion function x(t) of an object in spacetime is called the *Worldline* of the object.
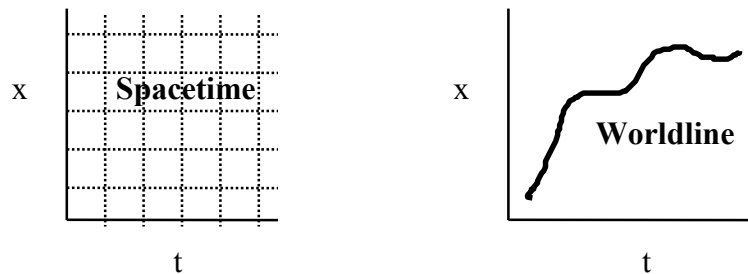
Spacetime is the *"fabric"* on which all events and motion in the world take place. What does your worldline look like in 4-dimensional spacetime?
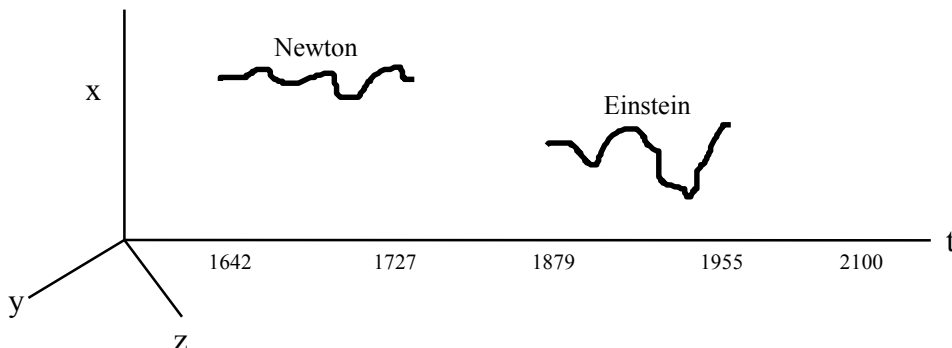
Constant Velocity ⟺ Straight Worldline

The kinematic trademark of *constant velocity* motion is that x(t) is a *linear function* of t , which implies that the *worldline* – the graph of x(t) – is a *straight line*.

Accelerated (changing velocity) motion is characterized by a *curved* worldline.

Here are the worldlines and the motion functions for three different kinds of motion:

At Rest	Constant Velocity	Constant Acceleration

$$x = x_o \qquad\qquad x = x_o + v_o t \qquad\qquad x = x_o + v_o t + \tfrac{1}{2} at^2$$
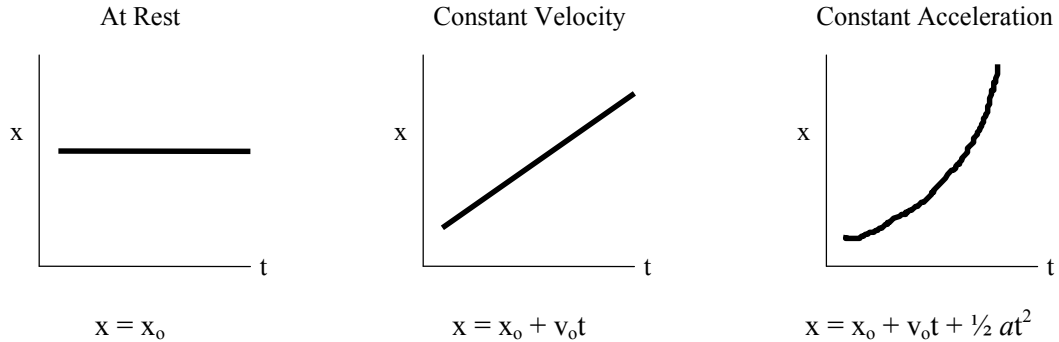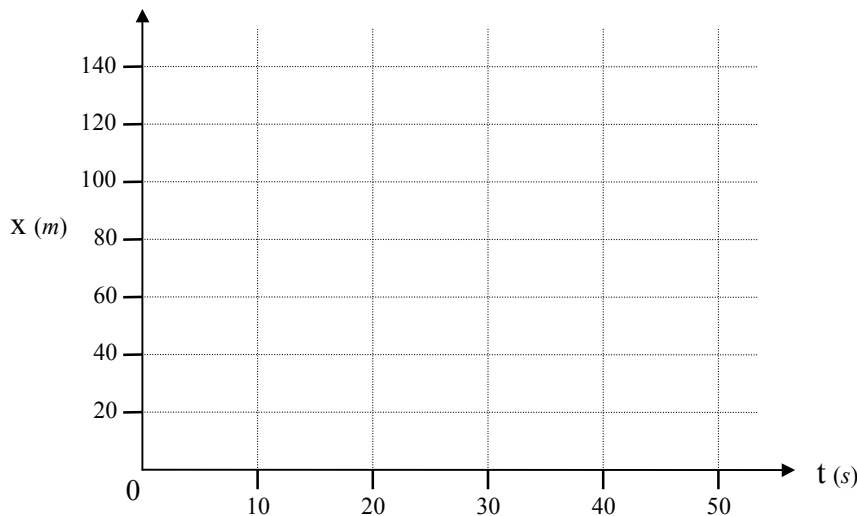
Exercise

A runner is running at a "steady pace". The following picture shows the location of the runner at five different readings of a stopwatch.

| 0.0 s | 10 s | | 25 s | 30 s | | 50 s |

| 30 *m* | 50 *m* | | 80 *m* | 90 *m* | | 130 *m* |
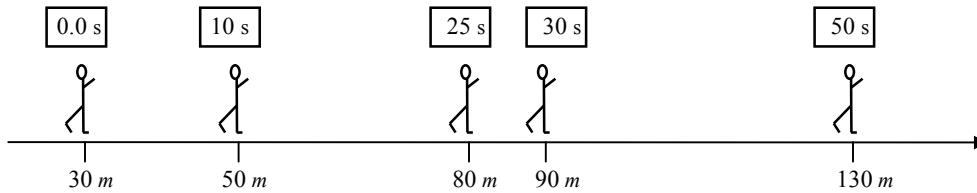
1. Plot the five *"events"* (t , x) and draw the *worldline* of the runner on the *spacetime* diagram:

2. What is the motion function x(t) of the runner?   Hint: x(t) = (*slope*) t + *intercept* .

x(t) = _____ .

3. What is the rate-of-change (derivative) of x(t) ?

dx/dt = _____ .

4. What is the velocity of the runner ?

v = _____ .

Part I.   The Motion Sensor

How Does it Work?

The *motion sensor* is specifically designed to measure the *motion function* x(t). This device emits a sound wave that bounces off the moving object and returns back to the sensor. The sound wave consists of a series of pulses emitted at a rate of 10 pulses per second, i.e. one pulse every 0.10 second.
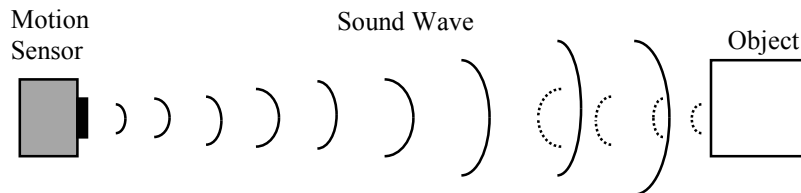

The sensor communicates with the computer via the Lab Pro Interface. The computer figures out the distance between the sensor and the object by knowing the speed of sound in air and the transit time of the pulse. The motion sensor is the sonar (sound wave) version of a radar (radio wave) device.

Thus, every *tenth of a second*, the motion sensor records the *position* of the object.
For example,

t (*s*)	0.00	0.10	0.20	0.30	0.40	0.50
x (*m*)	0.74	0.79	0.85	1.12	1.29	1.46

This table of x-t values defines the *motion function* x(t) – in *digital* form!   Each pair (t , x) of coordinates (a time and a place) is an *event*. Each event is a point in *spacetime*. The chain of event points is the *worldline* of the object.   The motion sensor is a "worldline-ometer" !

<u>Motion Sensor Fact</u>: The Distance (*m*) recorded by the motion sensor is equal to the distance between the front of the object and the metal disk (sound emitter) that is located just inside the front face of the sensor. In other words, the origin x = 0 of the x axis is the disk on the sensor. Do not touch the metal disk.
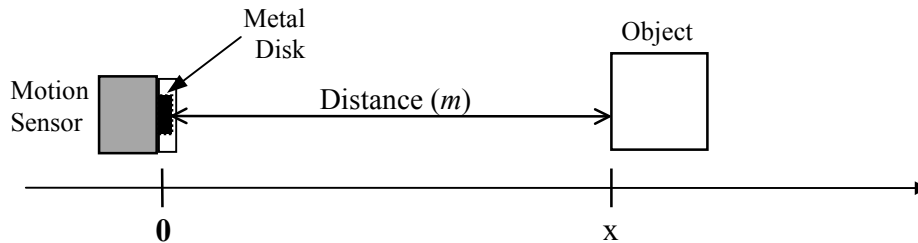
Collecting Data

Start the program *Logger Pro*. Open the file *Moving Along*. You will see an empty graph of Distance (*m*) versus Time (*s*) , i.e. x versus t . This is your **Spacetime Diagram**. To activate the motion sensor, press the *Space Bar* on the keyboard. You can also activate the sensor by clicking on the *Collect* button. Once activated, the table of data points (t , x) – the **Events** – will appear on the right side of your screen. The graph of the data points – the **Worldline** – will appear on the left side.

 Try to understand what the motion sensor is "seeing". Activate the sensor again and then move your hand toward and away from the sensor. Observe the worldline of your hand that is traced out in the graph window. Note that you can stop taking data by pressing the *Space Bar* or clicking on the *Stop* button.

Changing the Graph Scale

Suppose you want to collect data or display the graph of x(t) only for the first two seconds of the motion. You can change the time scale (horizontal axis) so that it goes from 0.0 *s* to 2.0 *s*. Here is one way to change the time scale: Click on the old number at the right end of the horizontal axis and type in the new number (2.0) and then press *Enter*. Note what happens. Now make the maximum distance 5.0 *m*. Activate the motion sensor, move your hand in front of the sensor, and see what appears on your graph.

Autoscale

The *Autoscale* command rescales the x and y axes so that the graph "fits" on the page. Collect some motion data. Look at the graph. Now click on the *Autoscale* icon |**A**   .   Note the new appearance of the graph.

Measuring Distances: Accuracy and Range of the Motion Sensor

Change the graph scales so that the time axis goes from 0 *s* to 10 *s* and the distance axis goes from 0 *m* to 2 *m* . Use a piece of cardboard or a large square plate as the "object". Such a large flat object serves as an effective target for the sound waves to hit and reflect from so that the motion sensor can readily see the object and monitor its position.

Using the meter stick to measure distance, place the object at a location that is 0.80 m away from the sensor (metal disk). Hold the object at this point and activate the sensor. Note the distance measured by the sensor. This distance can be read from the table on the right side of the screen. You can also find the distance directly on the graph: Click on the *Examine Icon* [x=?] at the top of the screen. Try this "Examine-the-Graph" method. It will be a handy feature for future experiments. Repeat for the distances listed in the table below and fill in the table. Find the percent difference between your (meter stick) value and the computer's (sonar range finder) value.

Statistics Fact: The *percent difference* between two numbers, such as 20 and 18, is $(20-18)/20 = 2/20 = 0.10 = 10\%$. You would pay $18 for a $20 shirt that was 10% off.

Distance x (m) Measured with Meter Stick	Distance x (m) Measured with Motion Sensor	Percent Difference
0.80		
0.50		
0.20		

All measuring instruments have built-in limitations on the range of values that they can measure. The motion sensor cannot be used to measure distances that are less than some minimum distance x_{min}. Try measuring distances that are less than 0.20 m and find the value of x_{min}. Report your value and explain how you found it.

$$x_{min} = \underline{\hspace{3cm}} m.$$

Part II. Kineseology and Kinematics

Kineseology $\equiv$ the study of the principles of mechanics and anatomy in relation to human movement. The science of sports.

Design Specs for 3-Part Motion

Consider moving from point A (x = 20 cm) to point B (x = 100 cm) to point C (x = 60 cm) in a time of 12 *seconds* as follows:

1. Move at a steady pace from A to B in 4 *seconds*.
2. Rest at B for 4 *seconds*.
3. Move at a steady pace from B to C in 4 *seconds*.

Plot this three-part motion on the following spacetime diagram. Note that t_o is some arbitrary time at which the motion begins. Write the value of the velocity (in *cm/s*) during each of the three 4-second intervals directly on each of the three corresponding line segments of the graph.

Spacetime Diagram
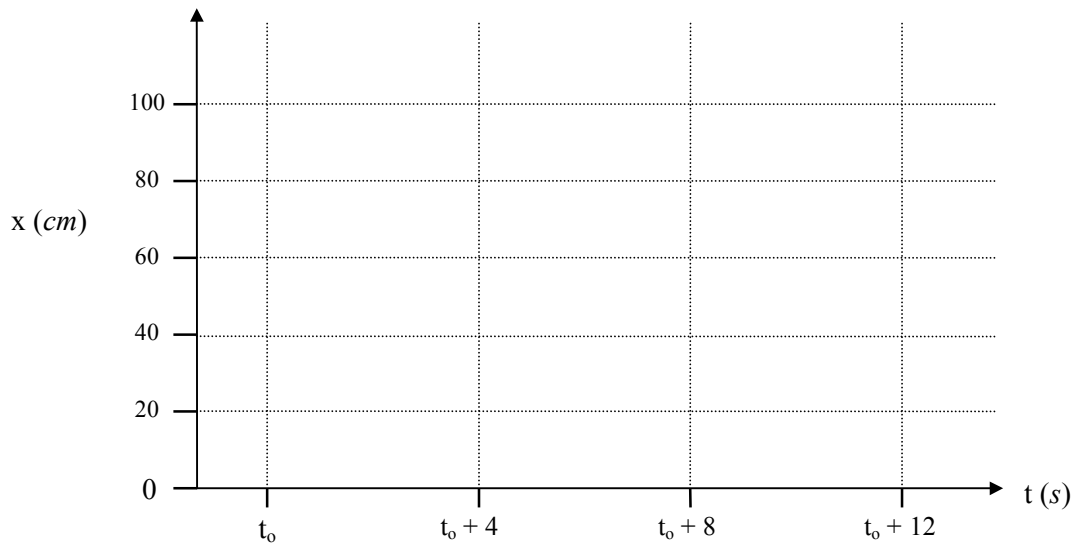
Creating the Worldline: Walking According to the Specs

First change the time scale to go from 0 to 15 *seconds* using the following new procedure: Click on "Experiment", go to "Data Collection", and type in the new time. Perform this three-part motion by walking alongside the track. Hold the cardboard object for the motion sensor to see. Your goal is to create an experimental worldline on your computer screen that matches the theoretical worldline that you graphed above. The match does not have to be "perfect", but the key kinematic features of your walking worldline (*positions*, *times*, *displacements*, *velocities*) should approximate the kinematic parameters of the design specs.

Hints: Use the meter stick to locate the three points (A,B,C) alongside the track. DO NOT WRITE or STICK TAPE on the TRACK. Practice moving at a constant velocity by pacing your walk so that you just cover the required distance during a 4-second count. Count the 4-second interval out loud (like a 4-beat measure in music): *"one-thousand one, one-thousand two, one-thousand three, one-thousand four"*.

PRINT the experimental x-t graph of your worldline. Label the graph "Walking Worldline". Print only the graph – not the data table. To print the graph only, click on File and select "Print Graph".

Note: You must write a short descriptive title at the top of <u>every</u> graph that you print in the physics lab. All professional scientific and engineering reports have such labeled graphs.

Questions

1. Is it possible for an object to move so that it produces a vertical line segment on the x-t graph? Explain.   Assume there are no astrophysical anomalies (blackholes, wormholes, ...) in the space.

2. The *Starship Enterprise* treks through space as follows: The starship starts at $(t, x) = (0,0)$ and moves at constant velocity from $t = 0$ to $t = 20$ s. At $t = 20$ s, the Enterprise is at $x = 10,000$ km at which point it enters a *"wormhole"*. While inside the wormhole, *the flow of time completely stops*. The starship exits the wormhole at $x = 30,000$ km and moves at a constant velocity of -500 km/s. Plot the worldline of the Enterprise (over the 60 s voyage) on the spacetime diagram:
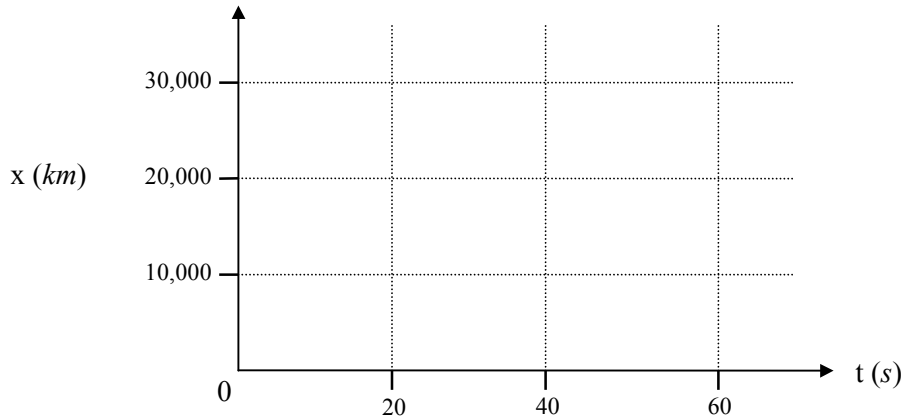
Part III. Motion of a Glider on an Air Track

You will now study the one dimensional "force-free" motion of a glider along an air track. The linear track defines the x axis. The glider "glides" on a layer of air making the motion almost "ideal" – free of the complicated effects of friction.

Leveling the Track

Gently lower the glider onto the track at different locations. If the glider always moves in the same direction, then the track is not level. To level a tilted track, you will need to raise or lower one end of the track by turning the screws (ever so slightly!) that are located on the bottom of one supporting leg. Since a level track is crucial for physics experiments, do not hesitate to check with your instructor about whether or not your track is level.

Launching the Glider

Launch the glider from the end of the track where the motion sensor is located so that it has a velocity of about 40 or 50 *cm/s*. Any value between 30 and 60 *cm/s* is perfectly fine. To give the glider this initial velocity, you can use your hand: push the glider very briefly and then let go. You could also use the rubber band attached to the track: push the glider against the rubber band and release. Practice this launching procedure. How fast is 50 *cm*/s ? Since the track is about 200 *cm* long, a glider moving with the velocity 50 *cm/s* will cover the length of the track in about 4 *seconds*. So as you count out loud "one thousand one, one thousand two, one thousand three, one thousand four", the glider should just make the trip from one end of the track to the other end. This observing-the-displacement / counting-the-time technique is a useful experimental procedure to estimate the velocity of any object.

Collecting the x(t) Data

Activate the motion sensor (press the *Space Bar* or click the *Collect* button). When you hear the clicking sounds (sonic vibrations) of the motion sensor, launch the glider. Stop the glider when it reaches the other end of the track – you only want to record the one-way trip down the track.

The Table of Time and Distance values on the right side of your computer screen stores the digital information on the *motion function* x(t) of your glider. The Distance-Time graph on the left side displays the Worldline of your glider in Spacetime.

Good Data Region

Remember the graphical trademark of constant-velocity motion: The x-t graph is a *straight line*. Make sure that your x-t graph is a straight line, except perhaps at the beginning and the end of the motion where the glider experiences "forces", such as the push of your hand at the beginning (launch point) of the track, or the collision with the rubber band bumper at the far end of the track. "Force" causes velocity to change and worldlines to curve. The region of your x-t graph that defines a smooth, straight line is the Good-Data Region.

If your x-t graph is not linear and/or has sharp intermittent "spikes" or "plateaus", then you will need to re-adjust your motion sensor and run the experiment again.
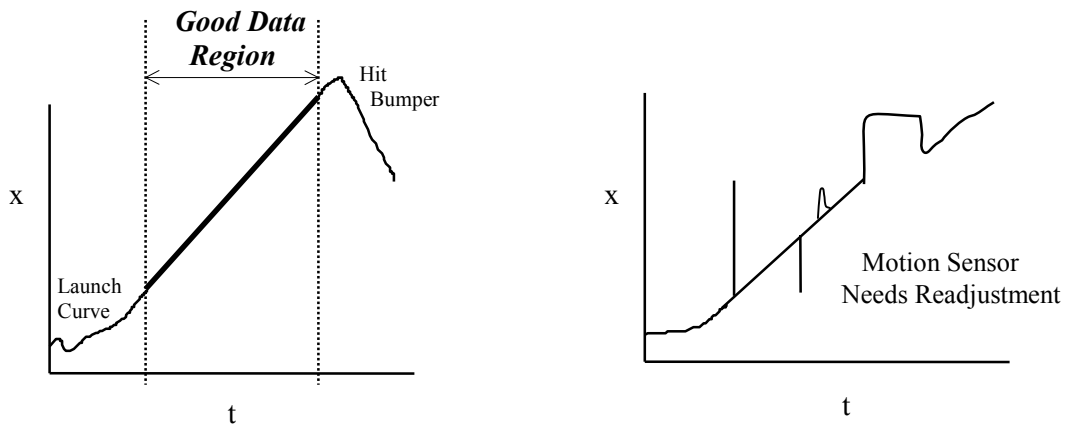
Whenever your x(t) graph contains "noise", the likely cause is that the motion sensor is not pointed in the optimal direction and therefore "sees" other objects besides the glider. The intermittent spikes and plateaus in the worldline occur every time the sensor detects some other (non-glider) object in its field of view. Try rotating the motion sensor so that the "cone" of emitted sound waves points slightly more in the upward direction – so that the wave does not reflect off the track and table surface.
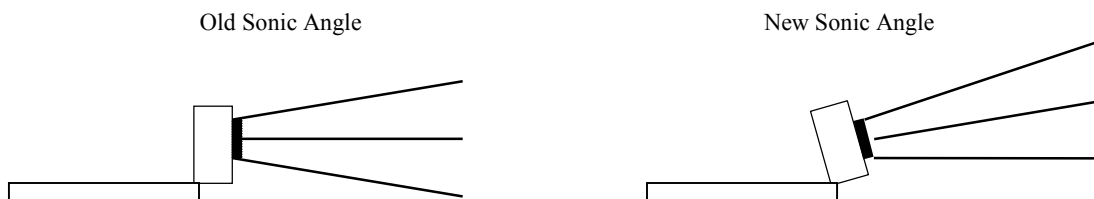
Velocity = Slope

If an object moves a distance Δx in a time Δt, then the average velocity of the object is $v \equiv \Delta x / \Delta t$. The ratio "Displacement over Time" is precisely equal to the "Rise over Run" of the x-t graph:

$$v \equiv \Delta x / \Delta t \;=\; slope\ of\ \mathrm{x(t)}\ line$$
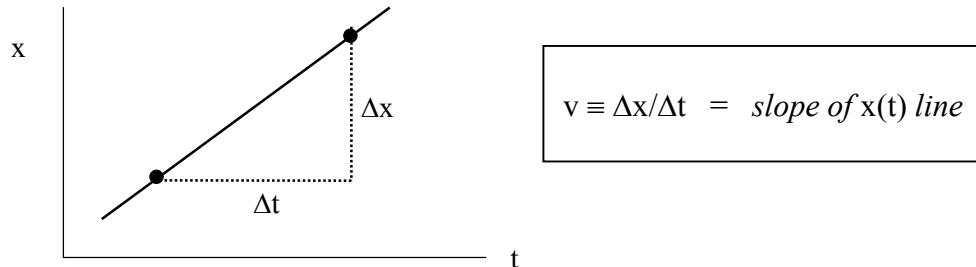
Find the slope of the worldline of your glider. Pick two points (*Event* 1 and *Event* 2) on the line. Make sure that the points are within the good data region and that the points are not too close to each other. Use the *Examine Icon* [x=?] to find the space-time coordinates of the two events: (t_1 , x_1) and (t_2 , x_2). Note that the *spatial* and *temporal* separation of the two events are $\Delta x = x_2 - x_1$ and $\Delta t = t_2 - t_1$, respectively.

PRINT the graph (not the table) displaying the worldline of your glider. Remember to write a short title on the graph. Label the "Good Data Region". Mark the two event points you chose on the line. Write the coordinate values next to each point. Show the calculation of the slope of the line *directly on the printed graph*. List your two events and report your *velocity = slope* value here:

Event 1: (_____ s , _____ m)   *Event* 2: (_____ s , _____ m)

$$v \;=\; slope\ of\ \mathrm{x(t)}\ line \;=\; \underline{\hspace{3cm}}\ m/s \,.$$

The Motion Function x(t) of your Glider

Find the equation that gives the position x(t) of your glider as a *function of time* t . State the time domain $t_{min} < t < t_{max}$ (good-data region) in which x(t) is valid.

*Motion Function*:   x(t) = _____ .

Time Domain: _____ s < t < _____ s .

Creating a Velocity Graph

So far you have a record of the motion of your glider in *tabular* form (t, x) and in *graphical* form (worldline). Given the position x(t) data, the calculation of the velocity v(t) is easy: divide the spatial interval dx = x(t+dt) – x(t) by the temporal interval dt = 0.10 *s* to find the velocity dx/dt at time t. The computer's result of calculating v(t) from x(t) is displayed in the third column of the data table window on your screen.   Look at this velocity column to see how v depends on t.

Create a graph of the *Velocity* of your glider as a function of *Time*. Here are the steps: Click on the Distance (meters) label of the vertical axis that appears on your Distance-Time graph. Select the Velocity option. A graph of Velocity versus Time should appear. Adjust the scale if necessary.

Locate the Good Data Region of your v-t graph. This is the region where the velocity is constant and thus the v-t graph forms a *flat line* – a straight line with zero slope. Your "flat" v(t) line will not look perfectly smooth. It is natural for the velocity to fluctuate up and down around some constant value. These random fluctuations are due to *experimental errors* such as friction, bumps and dirt on track, vibrating glider flag, and the computer's digital approximation to the smooth x(t) and v(t) functions.
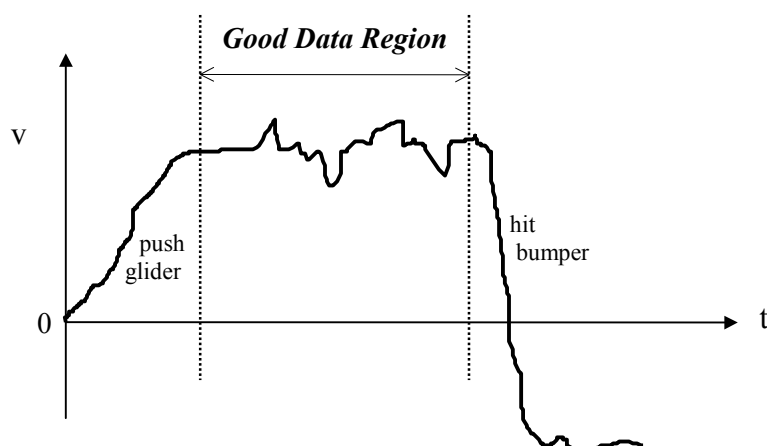
Make sure that your velocity graph is "flat" with slight "wiggles" that represent the up and down fluctuations in the velocity. You may have to change the velocity scale (vertical axis) in order to magnify the fluctuations. You could also try the "Autoscale" option.
PRINT your v-t graph.   Label the "Good Data Region".

Velocity = Average ±   Uncertainty

What is the constant value of the velocity that characterizes the "flatline" on your v-t graph ? In *statistics*, the _average value_ of the measured velocities represents the best estimate of the true value of the velocity, while the spread in the up and down fluctuations away from the average value – the deviations from flatness – determine the _uncertainty_ in the velocity. Here you will calculate the two statistical descriptors (average and uncertainty) of your v-t data.

Example: Calculating an *Average* and an *Uncertainty*

Suppose you measure the height H of a tree five times and find the following five values:

$$H = 5.4 \, m \,, \quad 6.0 \, m \,, \quad 5.9 \, m \,, \quad 5.4 \, m \,, \quad 5.8 \, m \,.$$

The average height is

$$H_{av} = (5.4 + 6.0 + 5.9 + 5.4 + 5.8 \,) / 5 \quad = \quad 5.7 \, m \,.$$

The uncertainty in the height, denoted by the symbol δH, can be quantified by the *"half-width spread"* in the values of H: $\delta H \equiv (H_{max} - H_{min})/2$:

$$\delta H = (6.0 - 5.4) / 2 \quad = \quad 0.3 \, m \,.$$

A scientist would report the results of measuring the height as " H (*average*) $\pm$ δH (*uncertainty*) " :

$$H = 5.7 \pm 0.3 \;\; m \,.$$

This statistical result says that, due to *experimental errors*, the "true value" of H most likely lies somewhere in the range between $5.7 - 0.3$ and $5.7 + 0.3$, i.e. $5.4 \, m < H < 6.0 \, m$. A graphical representation of this tree-height statistics is shown below:
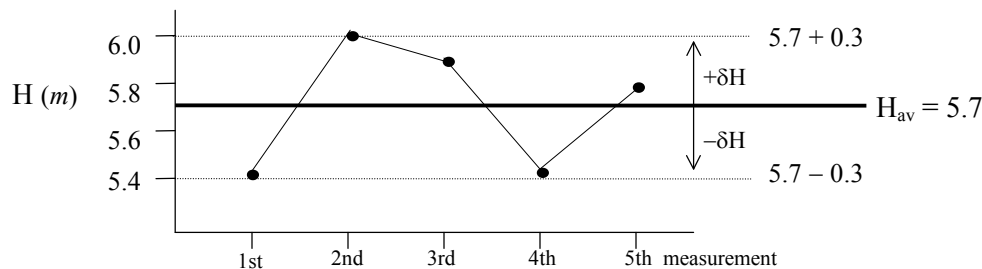
The Experimental Errors in measuring the height of the tree have several sources, such as judgement errors in reading the tape measure, elasticity of the tape, temperature effects, swaying of the tree due to wind, imprecise definitions of the "bottom" and the "top" of the tree, etc.

The *uncertainty* in the *measured results* is a reflection of the *errors* in the *measurement process*. In other words, the scatter or spread in the measured values depends on the equipment and procedure used to make the measurement. If you used one 12-inch ruler to measure the height of the tree, then the error in the measurement would be larger than that using the tape measure. This means that the uncertainty – the spread in the measured values of H – would be larger. If you used a laser beam, then the error and uncertainty would be smaller. For example, $\delta H = 1.2 \, m$ (ruler), $\delta H = 0.3 \, m$ (tape), $\delta H = 0.09 \, m$ (laser).

The Relative Uncertainty or "fractional uncertainty" provides the meaningful dimensionless measure of the "precision" or "accuracy" of the measurements. The relative uncertainty in the height of the tree in the above example is $\delta H/H_{av} = 0.3 \, m/5.7 \, m = 0.053$. You would conclude that the *"relative uncertainty is 5.3 %"*. If the average height were $57.0 \, m$ (instead of $5.7 \, m$), but with the *same* absolute uncertainty $0.3 \, m$, then you would know the height to a higher precision and confidence: $56.7 \, m < H < 57.3 \, m$. This higher precision is reflected in the smaller relative uncertainty $0.3/57.0 = 0.53$ %.

Five-Point Average of the Glider Velocity

Pick five points on your "flat" v-t line. Choose points that span the entire good-data region and occur at roughly equally-spaced intervals of time. Find the values of the velocity v at these points using the *Examine Icon* [x=?] or the Data Table. List your five measured values of v:

_____ _____ _____ _____ _____ *meters/second*.

Calculate the *average* value of v and the *uncertainty* in v:

Average ± Uncertainty: v   =   _____   ±   _____   *m/s* .

Relative Uncertainty =   _____ % .

Part IV.   Computational Physics

In the previous part, you calculated a slope value based on two points and an average value based on five points. Here you will learn how to use the computer to perform similar calculations on a larger scale and with higher precision by utilizing all the data points collected by your motion sensor. This involves two computational algorithms that are commonly used by physicists to analyze physical phenomena:

1. LINEAR ANALYSIS.   Finding best-fit lines and slopes.
2. STATISTICAL ANALYSIS.   Computing average values and standard deviations.

Linear Analysis.   Curve Fitting: Finding the Best x(t) Function

Change the Velocity-Time graph that appears in your window back to the Distance-Time graph.

To find the x(t) function that describes the trend in your x-t data points, you must first tell the computer which data points to analyze. In this experiment, the *good data region* is the constant velocity (straight sloping line) region of your x-t graph. To select this good data region, place the mouse at the left end of the region. Then press and hold the left mouse button down as you drag the mouse to the right end of the good data region. Stop when you reach the right end and release the button. The region that you have selected on the graph should now be highlighted in gray. In the table, the good data will appear shaded.

Having selected the good data for the computer to analyze, click on the *Curve Fit* icon [f(x)=?] and perform a "linear fit". The computer will draw the best-fit line through your x-t data points and will also display the equation of the line.   PRINT your graph showing the curve-fit box. Note that the computer gives you the equation of the line in the *generic* form "y = mx + b". In physics language, the equation of the line has the *kinematic* form   "x = vt + x₀" .

According to the computer's <u>linear analysis</u> of all the good x-t data recorded by your motion sensor, the motion function of the glider on the air track is

$$x(t) \;\; = \;\; \underline{\hspace{6cm}} \; .$$

Statistical Analysis.   Finding the Average Velocity and the Uncertainty

Change the Distance-Time graph that appears in your window to the Velocity-Time graph.

Select the good data region (where velocity is constant). Click on the *Statistics* icon [STATS]. A box will appear on your graph containing the statistical information. PRINT your graph showing the Stats Box. In addition to computing the <u>average value</u> (mean), the computer also computes the <u>standard deviation</u>.

The computer computes the average velocity as usual: Sum all the velocity values and divide by the number of values. To compute the standard deviation, the computer first finds the difference between each velocity value and the average. Each of these deviations is then squared.   The standard deviation is equal to the square root of the average of these squared deviations!

Note that the uncertainty in the velocity that you computed by hand in the previous part, namely $(v_{max} - v_{min})/2$, is a simple approximation to the standard deviation.

In physics, the *standard deviation* gives a precise measure of the *uncertainty* in the measured value of any physical quantity. If the average velocity is 0.479 *m/s* and the standard deviation is 0.0526 *m/s* , then you would report the measured velocity as <u>v = 0.48 ± 0.05 *m/s*</u> . *Relative Uncertainty*: An error of 0.05 *m/s* out of 0.48 *m/s* is 0.05/0.48 = 0.10, i.e. about 5 parts out of 50, and thus you would conclude that there is a 10% *uncertainty* in the velocity.

Significant Figures: Note that the value of the velocity 0.479 was rounded to the two "sig fig" value 0.48 because the uncertainty 0.0<u>5</u>26 lies in the hundredths place. If the uncertainty was 0.00526, then you would keep the third figure (thousandths place) in the velocity and report the measured value as 0.479 ± 0.005 *m/s* .

Bottom Line: The uncertainty in any measured quantity determines the number of significant figures that you should keep when you report the measured value of the quantity.

According to the computer's <u>statistical analysis</u> of all the good v-t data collected by your motion sensor, the velocity of the glider on the air track is

$$v \;\; = \;\; \underline{\hspace{4cm}} \;\; \pm \;\; \underline{\hspace{4cm}} \;\; m/s \; .$$

Relative Uncertainty = _____ % .

Conclusion

Measured Values of v

In this experiment, you have obtained the velocity of the glider using four different methods of analysis of the motion data. For the purpose of summary and comparison, rewrite the four values of velocity below:

$v = slope\ of$ x(t)   Your (two-point) Analysis: v   =

$v = average\ of$ v(t)   Your (five-point) Analysis: v   =

$v = slope\ of$ x(t)   Computer (many-point) Analysis: v   =

$v = average\ of$ v(t)   Computer (many-point) Analysis: v   =

Experimental Errors

List at least three sources of errors that you think are most responsible for the spread in your measured values of v.

What is the "True Velocity" of the Glider ?

As the "best estimate" of the velocity, find the average of your four values listed above. For the uncertainty in the velocity, find the half-width spread in your four values.

v   =   _____   ±   _____ m/s .

The relative uncertainty in v is   _____   % .

The "10% Rule" in the Physics Lab

The equipment and procedures in the Physics Labs are at least 10 % accurate.

This means that the percent difference between the experimental (measured) value and the theoretical (calculated) value of any physical quantity should be less than 10 %. Furthermore, the percent error (relative uncertainty) in any measured quantity should be less than 10 %.

If your team finds a percent difference or a percent error that is greater than 10% in any Physics lab this semester, then you need to consult with your lab instructor.