Acceleration

Part I. Uniformly Accelerated Motion: Kinematics and Geometry

*Acceleration* is the rate of change of velocity with respect to time: \( a = \frac{dv}{dt} \). In this experiment, you will study a very important class of motion called uniformly-accelerated motion. Uniform acceleration means that the acceleration is constant – independent of time – and thus the velocity changes at a constant rate. The motion of an object (near the earth’s surface) due to gravity is the classic example of uniformly accelerated motion. If you drop any object, then its velocity will increase by the same amount (9.8 m/s) during each one-second interval of time.

Galileo figured out the physics of uniformly-accelerated motion by studying the motion of a bronze ball rolling down a wooden ramp. You will study the motion of a glider coasting down a tilted air track. You will discover the deep connection between *kinematic* concepts (position, velocity, acceleration) and *geometric* concepts (curvature, slope, area).

A. The Big Four: \( t , x , v , a \)

The subject of *kinematics* is concerned with the description of how matter moves through *space* and *time*. The four quantities, time \( t \), position \( x \), velocity \( v \), and acceleration \( a \), are the basic descriptors of any kind of motion of a particle moving in one spatial dimension. They are the “stars of the kinema”. The variables describing space (\( x \)) and time (\( t \)) are the fundamental kinematic entities. The other two (\( v \) and \( a \)) are derived from these spatial and temporal properties via the relations \( v = \frac{dx}{dt} \) and \( a = \frac{dv}{dt} \).

Let’s measure how \( x \), \( v \), and \( a \) of your glider depend on \( t \). First make sure that the track is level. The acceleration of the glider on a horizontal air track is constant, but its value (\( a = 0 \)) is not very interesting. In order to have \( a \neq 0 \), you must tilt the track. Place two wooden blocks under the leg of the track near the end where the motion sensor is located. Release the glider at the top of the track and record its motion using the motion sensor. [Click on *Logger Pro* and open file *Changing Velocity 2*]. The graph window displays \( x \), \( v \), and \( a \) as a function of time \( t \). Your graphs should have the following overall appearance:
Focus on the good data region of the graphs where the acceleration is constant. To find this region, look for that part of the graphs where the x, v, a curves take on smooth well-defined shapes: $x = \text{parabola}, v = \text{linear}$ (sloping line), $a = \text{constant}$ (flat line). In the “bad data region”, the acceleration is changing because the glider is experiencing forces other than gravity, such as your hand pushing the glider or the glider hitting the bumper.

PRINT your x, v, a graphs and label the “good data region”. Have your instructor check your graphs and your good-data region before you move on to the next part of the lab.

B. Changing Velocity ⇔ Curving Worldline

Look at your x(t) graph and note: The worldline of your glider is curved!

Recall that in the constant-velocity lab, all graphs were straight.

Changing Velocity [1st change in v] is synonymous with a Curved Worldline [2nd change in x]:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}.$$  

Note that $d^2x/dt^2 = (dx/dt)/dt = d(\text{slope})/dt$ is the rate of change in the slope of x(t), i.e. the “curvature” of x(t). A straight worldline has a constant slope. A curved worldline has a changing slope. The amount of “bending” in a curve – the deviation from straightness – is measured by how much the slope changes.

The Importance of “Curvature” in Theoretical Physics

The gravitational force of the earth is the cause of the curved worldline of your glider. Remove the earth and the worldline would become straight. Two-Hundred and Fifty years after Newton, Einstein formulated his celebrated “Field Equations” of General Relativity which state the precise mathematical relationship between the amount of mass (the source of gravity) and the curvature of spacetime. Force causes x(t) to curve. Mass causes spacetime to warp.

Measuring the “Curvature” of Your Glider’s Worldline

Click on the Slope Icon [m=?] and find the slope dx/dt at five different points on the x(t) curve that are separated by 0.5 second intervals. Fill in the table below. Note that dx/dt is equal to the instantaneous velocity v of the glider. Make sure that the points are within the good-data region. Find the change-in-slope Δv by subtracting your neighboring values of slope v. Find the rate of change in slope Δv/Δt by dividing each of your Δv’s by Δt = 0.50 s.
"Curvature and Acceleration" Table

<table>
<thead>
<tr>
<th>time t (s)</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope of x(t): v = dx/dt (m/s)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in Slope: ∆v (m/s)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rate of Change in Slope: ∆v/∆t (m/s²)</td>
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<td></td>
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</tr>
</tbody>
</table>

Note how the slope increases with time. This change in slope tells you how much the x(t) parabola “curves away” from a straight line. In contrast, the change-in-slope should be approximately constant. Can you conclude that the acceleration a of the glider is constant (within a 10% error margin)? What is the value of a (to two significant figures) based on this analysis of the slope change or “curvature” of x(t)?

Rate of Change in Slope of x(t):  

\[
a = 0. \quad _____ \quad _____ \quad \text{m/s}^2.
\]

Now find the value of the acceleration of your glider directly by looking at the entries in the Acceleration column of the motion-sensor data table. Like all experimental quantities, the values of a(t) will fluctuate around some average value. Estimate the average value (to two significant figures) simply by looking at the table and noting the number(s) that occur most often.

Average of a(t):

\[
a = 0. \quad _____ \quad _____ \quad \text{m/s}^2.
\]

Compare your two values of a.

% Difference = _____ %. If % Diff > 10%, then see your instructor.

C. Acceleration = Slope. Displacement = Area.

In the previous section, you found the value of a from the x(t) graph via the relation \( a = d^2x/dt^2 \). In this section, you will find the value of a from the v(t) graph via the relation \( a = dv/dt \).

Pick two points on the v(t) line within the good-data region that are not to close to one another. Find the values of t and v at these points using the Examine Icon [x=?] or from the data table. Also find the position x of the glider at these same two times.

\[
\begin{align*}
t_1 &= \quad _____ \quad s \quad \quad v_1 = \quad _____ \quad \text{m/s} \quad \quad x_1 = \quad _____ \quad \text{m}. \\
t_2 &= \quad _____ \quad s \quad \quad v_2 = \quad _____ \quad \text{m/s} \quad \quad x_2 = \quad _____ \quad \text{m}.
\end{align*}
\]
PRINT your v(t) graph window (not x and a). Label the points 1 and 2 with your pen. Write the coordinate values \((t_1, v_1)\) and \((t_2, v_2)\) next to each point. Calculate the following two geometric properties of the v(t) graph:

1. Slope of the line.  
2. Area under the line between \(t_1\) and \(t_2\).

Show your calculations (rise-over-run, base-times-height etc.) directly on your printed graph. Report your slope and area results here:

\[ \text{Slope of v(t) line} = \frac{\text{rise}}{\text{run}} \ (m/s)/s. \quad \text{Area under v(t) line} = \text{base} \times \text{height} \ (m/s) \times s. \]

Two Key Observations: 1. The ratio \(dv/dt\) is the rise (dv) over the run (dt) of the line.  
2. The product \(vdt\) is the area of the rectangle of base \(dt\) and height \(v\). Conclusion:

\[ a = \frac{dv}{dt} \quad \Rightarrow \quad \text{Acceleration } a = \text{Slope of v(t) graph}. \]
\[ dx = vdt \quad \Rightarrow \quad \text{Displacement } \Delta x = \text{Area under v(t) graph}. \]

You already found \(a\) (from the \(a\) values in the data table). Write this value of \(a\) again in the space below. From your measured values of \(x_1\) and \(x_2\) (listed above), you can find the displacement of the glider: \(\Delta x = x_2 - x_1\), i.e. the distance moved by the glider during the time interval from \(t_1\) to \(t_2\).

\[ a = \frac{\text{rise}}{\text{run}} \ m/s^2. \quad \Delta x = \text{base} \times \text{height} \ m. \]

Compare this value of \(a\) with your value of “Slope of v(t) line”. Compare this value of \(\Delta x\) with your value of “Area under v(t) line”.

\[ \% \text{ diff between } a \text{ and slope} = \text{______}\%. \quad \% \text{ diff between } \Delta x \text{ and area} = \text{______}\%. \]

Physics and Calculus. The problem of finding slopes and areas is the essence of the whole subject of Calculus. Newton invented Calculus for to understand Motion. In Calculus, “finding slopes (accelerations)” and “finding areas (displacements)” are inverse operations called “differentiation” and “integration”, respectively.
D. Computational Physics : Finding the “Best Value” of $a$

Let’s use the full computational power of the computer to find the acceleration of the glider by analyzing all the data collected by the motion sensor.

**Statistical Analysis of $a(t)$**

Select the good-data region of your $a(t)$ graph. Remember the data selection procedure: click and drag from the left end to the right end of the good region. To perform a statistical analysis of the selected data, click on the Statistics Icon [STAT]. The computer will find the average (mean) value and the standard deviation. Recall that the average value provides the best estimate of the “true value” of the quantity, while the standard deviation is the uncertainty – the spread in the measured values around the average due to the experimental errors.

The averaging procedure smoothes out the up and down fluctuations in the measured $a(t)$. There are several sources of experimental errors that cause the acceleration of the glider to fluctuate over time. These errors include a bumpy track, dirt on track, a bent glider, dirt on glider, surface friction, air friction, and the fact that the motion sensor approximates the continuity of motion – the smooth flow of time – by collecting and analyzing data in discrete time steps.

\[
\text{Average of } a(t) \pm \text{Uncertainty: } a = \text{______________} \pm \text{______________} \text{ m/s}^2.
\]

**Linear Analysis of $v(t)$**

The acceleration of the glider is equal to the slope of the $v(t)$ line. Select the good-data region of your $v(t)$ graph. Click on the Curve-Fit Icon [$f(x)=?$] and perform a “Linear Fit” to find the best-fit line through the $v$-$t$ data points. The computer will give the equation of the line as $y = mx + b$, which in velocity-time language is $v = at + v_o$. The slope of the best-fit line gives the “best value” of $a$.

\[
\text{Equation of Best-Fit } v(t) \text{ Line: } v(t) = \text{______________________________}.
\]

\[
\text{First Derivative (Slope) of } v(t): \frac{dv}{dt} = \text{______________} \text{ m/s}^2.
\]

**Quadratic Analysis of $x(t)$** (Optional Extra Credit)

\[
\text{Equation of Best Fit } x(t) \text{ Curve: } x(t) = \text{______________________________}.
\]

\[
\text{Second Derivative (Curvature) of } x(t): \frac{d^2x}{dt^2} = \text{______________} \text{ m/s}^2.
\]
Natural “Gee Units”

Since we live in the gravitational field of the earth and feel its strength daily, it is convenient to adopt the special number \( g = 9.8 \text{ m/s}^2 \) as the natural unit of acceleration and express all other accelerations relative to \( g \). For example, an acceleration \( a = 4.9 \text{ m/s}^2 \) is \( a = 0.50 \text{ g} \). A jet pilot that experiences “8 gees” is moving with an acceleration \( a = 8 \times 9.8 \text{ m/s}^2 \). What is the best value of the acceleration of your glider in \( \text{m/s}^2 \) units and in \( \text{g units} \)?

Note: The three “best values” of the acceleration based on the computer analysis of \( a(t), v(t), \) and \( x(t) \) should be the “same” (percent difference should be less than 5%). You can average the slightly different values to find the one best value.

\[
a = \underline{\text{___________}} \text{ m/s}^2 \quad = \quad \underline{\text{___________}} \text{ g}.
\]

Part II. The Physics of Free Fall

Consider an object of mass \( m \) that is released from rest near the surface of the earth. After a time \( t \), the object has fallen a distance \( y \) and is moving with velocity \( v \). The free-fall equations relating \( y, t, \) and \( v \) are

\[
y = \frac{1}{2} gt^2, \quad v = gt, \quad v^2 = 2gy,
\]

where \( g = 9.8 \text{ m/s}^2 \) is independent of \( m \).

In this experiment, you will test these important properties of free-fall motion by studying the motion of a glider on a tilted air track. Strictly speaking, free fall refers to the vertical motion of a body that is free of all forces except the force of gravity. A body moving on a friction-free inclined track is falling freely along the direction of the track. It is non-vertical free fall motion. The track simply changes the direction of the fall from vertical to “diagonal”. This diagonal free fall is a slowed-down and thus easier-to-measure version of the vertical free fall. The acceleration along the track is the diagonal component of the vertical \( g \). This acceleration depends on the angle of incline. It ranges from 0 \text{ m/s}^2 \) at 0° (horizontal track) to 9.8 \text{ m/s}^2 \) at 90° (vertical track). In other words, the track merely dilutes gravity. A frictionless inclined plane is a “gravity diluter”. We will denote the acceleration \( a \) of the glider falling down the track as the diluted value of the full-strength \( g \):

\[
a = g(\text{diluted}).
\]
A. Experimental Test of the Squared Relation  \( d \sim t^2 \)

In theory, the worldline of the glider is a parabola. Hence the distance \( d \) traversed by the glider along the track is proportional to the square of the time elapsed (after starting from rest). This means that if you double the time, \( t \rightarrow 2t \), then the distance will quadruple, \( d \rightarrow 4d \). More specifically, if it takes time \( t_1 \) to move distance \( d_1 \) and time \( t_2 \) to move distance \( d_2 \), then the proportionality \( d \sim t^2 \) implies the following equality of ratios: \( \frac{d_2}{d_1} = \left( \frac{t_2}{t_1} \right)^2 \). This ratio relation says if \( t_2 = 2t_1 \), then \( d_2 = 4d_1 \).

Start with the titled track with two blocks under the end of the track. Use a stopwatch – not the motion sensor – to measure the time it takes the glider, starting from rest, to move a distance of 25 cm down the track. Repeat three more times and find an average time. Next measure the time it takes, starting from rest, to move a distance of 100 cm.

<table>
<thead>
<tr>
<th>Average Time</th>
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<tbody>
<tr>
<td>t (d = 25 cm)</td>
</tr>
<tr>
<td>t (d=100 cm)</td>
</tr>
</tbody>
</table>

Are your experimental results consistent with the theoretical relation \( d \sim t^2 \)? Explain carefully by constructing ratios. Hint: compare \( \frac{d_2}{d_1} \) and \( \left( \frac{t_2}{t_1} \right)^2 \) as discussed in the above paragraph.

Calculate the value of the acceleration \( a = g(diluted) \) of the glider along the track direction from your measured values of \( d \) and \( t \). Show your calculation.

For \( d = 25 \text{ cm} \), \( a = \) \underline{ } \text{ m/s}^2.

For \( d = 100 \text{ cm} \), \( a = \) \underline{ } \text{ m/s}^2.
B. Experimental Test of $v^2 \sim H$

*Physics Fact:* The speed $v$ of an object, starting from rest and falling down the frictionless surface of an inclined plane, depends only on the *vertical height* $H$ of the fall and not the length of the incline. Furthermore, the *square* of the velocity is proportional to the height: $v^2 \sim H$. This squared relation implies that the speed will *double* if the height *quadruples*.


Since you are testing the proportionality, $v^2 \sim H$, and not the equality $v^2 = 2gH$, you only need to study how $v$ depends on the *number of blocks* that you stack vertically to elevate the track. The height $H$ can be measured in *dimensionless units*, simply as the “number of blocks”.

Place one block ($H = 1$) under the motion-sensor end of the track. Position the glider at the point that is 20 cm away from the sensor. Release the glider from rest and measure its velocity $v$ (using the sensor) when it is 100 cm away from the sensor. Repeat three more times and find an average velocity. Now quadruple the height by placing four blocks ($H = 4$) under the end. Once again, release the glider at 20 cm and measure its velocity at 100 cm.


<table>
<thead>
<tr>
<th>Average Velocity</th>
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</thead>
<tbody>
<tr>
<td>$v$ (H = 1)</td>
</tr>
<tr>
<td>$v$ (H = 4)</td>
</tr>
</tbody>
</table>

Do your experimental results support the theoretical relation $v^2 \sim H$? Explain carefully by *constructing ratios.*
C. Experimental Test of the Universality of $g$

One of the deepest facts of Nature is that the acceleration of an object due to gravity does not depend on the size, shape, composition, or mass of the object. Use two blocks to incline the track. Use the motion sensor to record the motion of the glider as it falls freely down the track. Find the acceleration of the glider by averaging the $a$ versus $t$ data. Remember to carefully select the region of “good data” on the graph before you analyze the data. To find the average of the $a(t)$ values, click on the statistics icon [STATS]. Report your results in the table below. Include the uncertainty (standard deviation) in each $a$ and the range of each $a$. For example, if the statistical analysis of the acceleration data gives the average value 0.347 m/s$^2$ and the standard deviation 0.021 m/s$^2$, then you would report your measured value of acceleration to be $0.35 \pm 0.02$ m/s$^2$, or $35 \pm 2$ cm/s$^2$. The range of this $a$ is $33 \rightarrow 37$ cm/s$^2$.

Add two weights or “metal donuts” (one on each side of the glider) and measure the acceleration. Add four weights (two on each side) and measure the acceleration.

<table>
<thead>
<tr>
<th>$m$ (kg)</th>
<th>$a \pm$ uncertainty (cm/s$^2$)</th>
<th>Range of $a$ (cm/s$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\pm$</td>
<td>$\rightarrow$</td>
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<tr>
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<td>$\pm$</td>
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<tr>
<td></td>
<td>$\pm$</td>
<td>$\rightarrow$</td>
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</tbody>
</table>

Do your experimental results support the deep principle that $g$, or equivalently $g(diluted)$, is independent of mass? Remember that your measured values of $a$ are equal to $g(diluted)$. So the big question is: Are your three measured values of $a$ equal?

When are two experimental values equal?

To answer this question, the role of uncertainty is vital. A measured value such as $15 \pm 2$ is really a range of numbers $13 \rightarrow 17$. Two experimental values are equal if and only if their ranges overlap. Suppose you are given two rods (A and B) and measure their lengths to be $L_A = 15 \pm 2$ cm and $L_B = 18 \pm 3$ cm. Since the two ranges overlap, $13 \rightarrow 17$ and $15 \rightarrow 21$, you can conclude that these two rods are equal in length. A range diagram provides an excellent visual display of the experimental values of measured quantities. The following range diagram for $L_A$ and $L_B$ clearly exhibits the amount of overlap:
Plot your three measured values of $a$ on the following range diagram:

![Range Diagram](image-url)

Now you can rigorously answer the important question: Do your measured values of $a$ provide an experimental proof of the deep principle that the acceleration of gravity is independent of mass? Explain.

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**Einstein, Curved Space, Black Holes, Warp Drive**

In a gravitational field, all bodies fall with the same acceleration. We have said this is a deep law of nature. Indeed, Einstein used this law as the basis for his general theory of relativity. All bodies fall in the same way because they are merely “coasting along” the same “downhill” contours of the curved space that they happen to occupy. Einstein’s field equations tell you precisely how to calculate the curvature of four-dimensional space-time. *Gravity is not a force – it is the shape of space.* The idea that “gravity is curvature” is the basis for bending light, gravity waves, black holes, and wormholes.

In essence, your experimental proof that “$g$ is independent of $m$” is a proof of the existence of black holes and gravity waves! When warp drive is invented, you will appreciate that it is a consequence of the universality of $g$. 

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Part III. Designing a Diluted-Gravity System

In vertical free fall, an object released from rest moves about 45 m in 3.0 s. You need to slow this motion – dilute gravity – so that the object only moves 1.5 m in 3.0 s. Your goal is to find how much the track needs to be tilted to achieve this slowed-down motion. First work out the theory and then perform the experiment.

The Theory

Architecture Diagram. \( H = \) height of blocks. \( L = \) distance between track legs. \( \theta = \) angle of incline.

![Architecture Diagram](image)

Acceleration Diagram. \( g = \) full strength gravity. \( a = \) diluted gravity.

![Acceleration Diagram](image)

Three Step Solution

1. Measure the length \( L \) between the “legs” of the track (see architecture diagram).
   Note: \( L \neq \) length of whole track.
   \[ L = \ldots \text{ m}. \]

2. Calculate the acceleration that the glider must have in order to satisfy the Design Specs: “glider released from rest and moves 1.5 m in 3.0 s”.
   \[ a = \ldots \text{ m/s}^2. \]
3. Calculate the height $H$ of the blocks that is necessary to achieve this amount of acceleration. 
   Hint: The architecture diagram shows $\sin \theta = H/L$. The acceleration diagram shows $\sin \theta = a/g$.

$$H = \______________ \text{ cm}.$$ 

The Experiment

1. Again, make sure that the track is level. Raise the end of the track by the height $H$ predicted above. To achieve this value of the height (to within tenths of a centimeter), you will most likely need to place one or more of the thin square metal plates on top of the wooden blocks. Measure the thickness of the plate with the Vernier caliper.

2. Release the glider from rest. Use a stopwatch to measure the time $t$ it takes the glider to move 1.5 m along the track. To achieve greater accuracy in timing, measure the 1.5 m distance from the elastic cord (bumper) that the glider hits at the lower end of the track. Seeing/hearing the glider hit the cord provides a well-defined signal for you to stop the watch.

   Repeat this measurement five times. List your five values of $t$ below and compute the average time and the uncertainty in the time. Estimate the uncertainty (deviation from average) from the “half-width spread” in your five values of time: $Uncertainty = (t_{\text{max}} - t_{\text{min}})/2$.

   \[
   \begin{array}{c|c|c|c|c}
   \text{t (s)} & | & | & | & |
   \end{array}
   \]

   $$t = \______________ \pm \______________ \text{ s}.$$ 

3. Compare this measured value of time with the design goal of $t = 3.0 \text{ s}$. What is the percent difference? Clearly show whether or not the theoretical time of 3.0 seconds falls within your experimental range? If not, what source(s) of error could account for the discrepancy?