

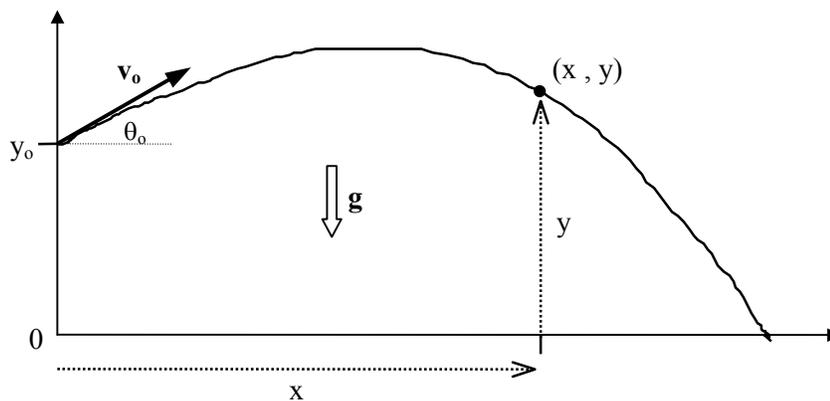
Team: _____

Projectile Motion

So far you have focused on motion in one dimension: $x(t)$. In this lab, you will study motion in two dimensions: $x(t)$, $y(t)$. This 2D motion, called “projectile motion”, consists of a ball projected with an initial velocity in the earth’s gravitational field.

Basic Principles

Consider launching a ball with an initial velocity v_0 near the surface of the earth where the acceleration of gravity is g .



The position of the ball is given by the coordinates (x, y) . The position of the ball depends on *time* t . The motion of the ball is defined by the *motion functions*: $x(t)$, $y(t)$. Note that at time $t = 0$, the ball is launched from the point $(x, y) = (0, y_0)$ with the velocity v_0 . The initial velocity vector v_0 has magnitude v_0 and direction θ_0 . Given the position and velocity of the ball at time zero, can we figure out the position of the ball for *all future time*? In short, here is the fundamental “*Kinematic Quest*” of projectile motion:

Given: The Initial Conditions: y_0, v_0, θ_0 .

Find: The Motion Functions: $x(t), y(t)$.

The quest to find how $x(t)$ and $y(t)$ depend on t is greatly simplified by the following facts, first discovered by Galileo:

The horizontal $x(t)$ and vertical $y(t)$ motions are completely independent of each other.

$x(t)$ = constant-velocity motion. $y(t)$ = constant-acceleration motion.

Analyzing the whole complicated motion as a superposition of manageable parts is a paradigm of modern theoretical physics.

The theory of projectile motion goes as follows. The general motion functions for any kind of *uniformly-accelerated motion* in two dimensions are

$$x(t) = x_0 + v_{0x}t + \frac{1}{2} a_x t^2 .$$

$$y(t) = y_0 + v_{0y}t + \frac{1}{2} a_y t^2 .$$

Projectile motion is a special case of uniformly-accelerated motion. Near the surface of the earth, the acceleration of gravity points downward and has magnitude 9.8 m/s^2 and therefore $(a_x, a_y) = (0, -9.8 \text{ m/s}^2)$. Substituting these special “earth gravity values” of a_x and a_y into the general motion functions $x(t)$ and $y(t)$ displayed above, and also setting x_0 equal to zero for convenience, gives the following motion functions for any object projected in the earth’s gravitational field:

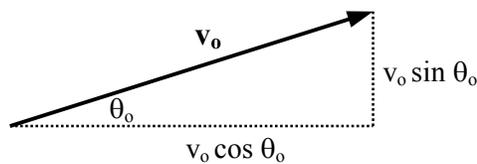
The Projectile Motion Equations

$$x(t) = v_{0x} t$$

$$y(t) = y_0 + v_{0y} t - 4.9 t^2$$

These equations tell you everything about the motion of a projectile (neglecting air resistance). If you know the conditions (y_0, v_{0x}, v_{0y}) at $t = 0$, then these equations tell you the position $(x(t), y(t))$ of the projectile for *all future time* $t > 0$. Make sure you understand *The Projectile Motion Equations*. They will be used in all future parts of this lab.

Note: In terms of the initial launch angle θ_0 , the components (v_{0x}, v_{0y}) of the initial velocity vector \mathbf{v}_0 are $v_{0x} = v_0 \cos \theta_0$ and $v_{0y} = v_0 \sin \theta_0$.



Exercise

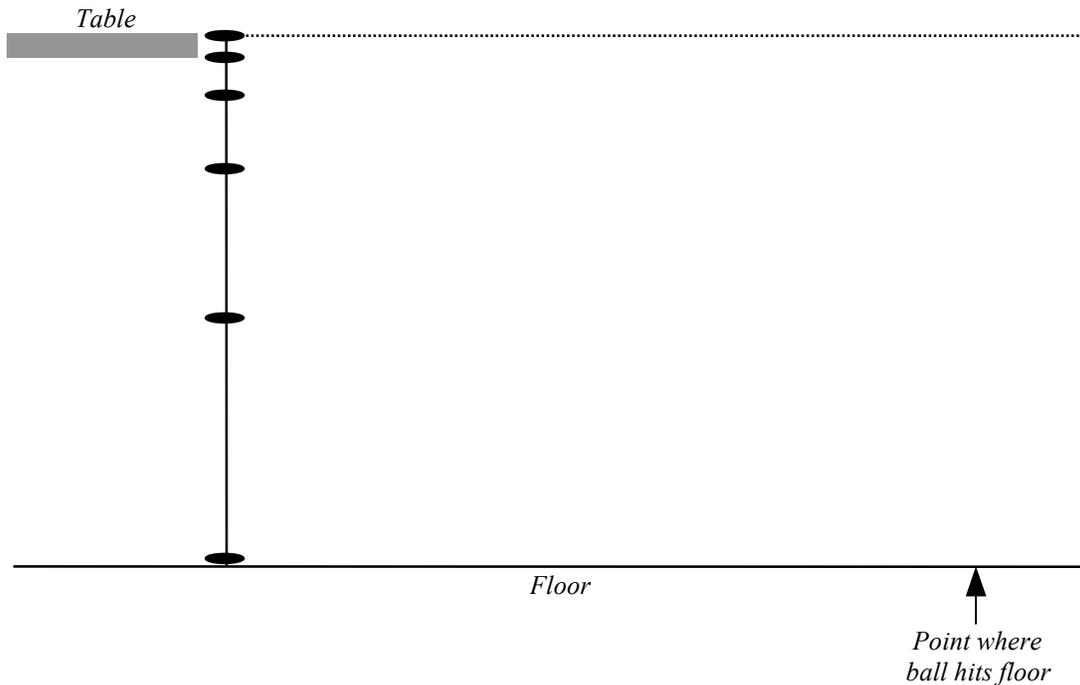
The initial ($t = 0$) launch parameters of a projectile are $y_0 = 3.6 \text{ m}$, $v_0 = 8.9 \text{ m/s}$, $\theta_0 = 54^\circ$. Where is the projectile at time $t = 1.2 \text{ seconds}$?

$x = \underline{\hspace{2cm}} \text{ m.} \qquad y = \underline{\hspace{2cm}} \text{ m.}$

Part I. An Illustration of the Independence of $x(t)$ and $y(t)$

Roll the plastic ball off the edge of the table. At the instant the ball leaves the table, drop a coin from the edge of the table. Listen for the ball and the coin to hit the floor. Try rolling the ball faster off the table. Summarize your observations:

The picture below shows the position of the dropped coin at six different times. The time interval between consecutive positions is the same. The point at which the ball hits the floor is also shown. Use a ruler to draw horizontal and vertical grid lines – *spaced just right* – that will allow you to plot the precise location of the ball at six different times. Neglect air friction. Use your six points to sketch the path of the ball as it moves through the air from table to floor.



Part II. The Ball Launcher. Finding the Initial Speed.

In this lab, you will use a “projectile machine” to give the ball an initial velocity, i.e. to project the ball in a certain direction with a certain speed. This ball launcher is a spring-loaded “cannon”. Note that you can change the angle θ_0 of the launch (the direction of the \mathbf{v}_0 vector) by tilting the launcher (loosen and tighten the nut). Note that the value of θ_0 is measured relative to the horizontal. A protractor on the side of the launcher specifies the numerical value θ_0 . Tilt the launcher so that it is set for a $\theta_0 = 30^\circ$ launch. Place the plastic ball inside the barrel of the launcher. Use the plunger rod to *slowly* push down on the ball until you hear and feel the first “click”. At this first setting, the spring is locked into a state of minimum compression.

CAUTION: DO NOT compress the spring beyond the first setting.

CAUTION: Always make sure that the launcher is aimed in a SAFE direction, away from people and objects.

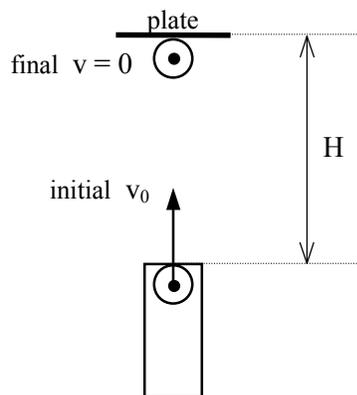
When it is safe, pull the lever that launches the ball. Observe the projectile motion.

What is the Initial Speed of the Projectile?

Here you will find the value of v_0 = the speed of the ball as it exits the launcher. This is known as the “muzzle velocity” of the cannon. First note that the point at which the ball exits the launcher (leaves the spring) is marked as a dot at the center of the small circle that appears on the side of the launcher. This dot represents the “*center of mass*” of the ball (circle). Since the initial speed of the ball is determined by the force of the spring (gravity has a negligible effect), the value of v_0 is constant, independent of the tilt of the launcher.

Tilt the launcher so that it points upward in the vertical direction ($\theta_0 = 90^\circ$). Launch the ball and observe how high the ball rises. By measuring the maximum height H , you can determine the launch speed v_0 .

Here is an experimental technique to determine H . On your table is a vertical rod assembly with a small metal plate attached to the rod. Position the plate directly above the launcher near the point where the ball reaches its maximum height. Adjust the plate up or down so that the launched ball barely hits or barely misses the plate. Launch the ball several times, each time “*fine tuning*” the vertical position of the plate (slightly up/down) until you are confident ($\pm 1\text{ cm}$) in the location of the maximum height. Measure the distance from the top of the ball at the launch point to the bottom of the plate.



Maximum Height: $H = \underline{\hspace{2cm}} m.$

From this measured value of H , compute the initial speed v_0 of the ball. Show your calculation.

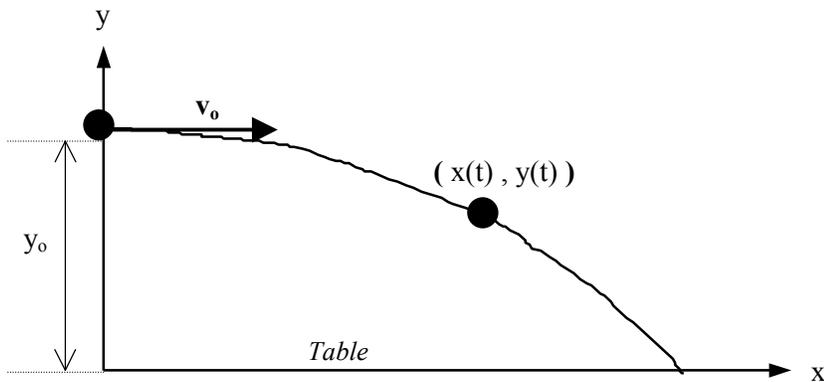
Initial Speed: $v_0 = \underline{\hspace{2cm}} m/s.$

Part III. Discovering the Parabola

Geometric trademark of projectile motion: Projectile Path = Parabolic Curve.

Launch the ball at some angle and observe the *curved path* traced out by the ball as it moves through space. It is difficult to map out the exact shape of the path when you only have about *one second* to make the observation! The path is definitely some kind of curve, but how do you know it is a parabola? Can you rule other shapes: semi-circular arc, or an oval curve, or a cubic curve, or an exponential curve, or a piece of a sine curve ?

Let's explore the special case of a *horizontal* launch. Set the launch angle to be $\theta_0 = 0^\circ$.



The Initial Conditions

Measure the initial height y_0 of the ball. *Note:* measure from the table surface to the *bottom* of the ball (see picture). Record your previously measured value of the initial speed v_0 of the ball.

$y_0 = \underline{\hspace{2cm}} m.$ $v_0 = \underline{\hspace{2cm}} m/s.$

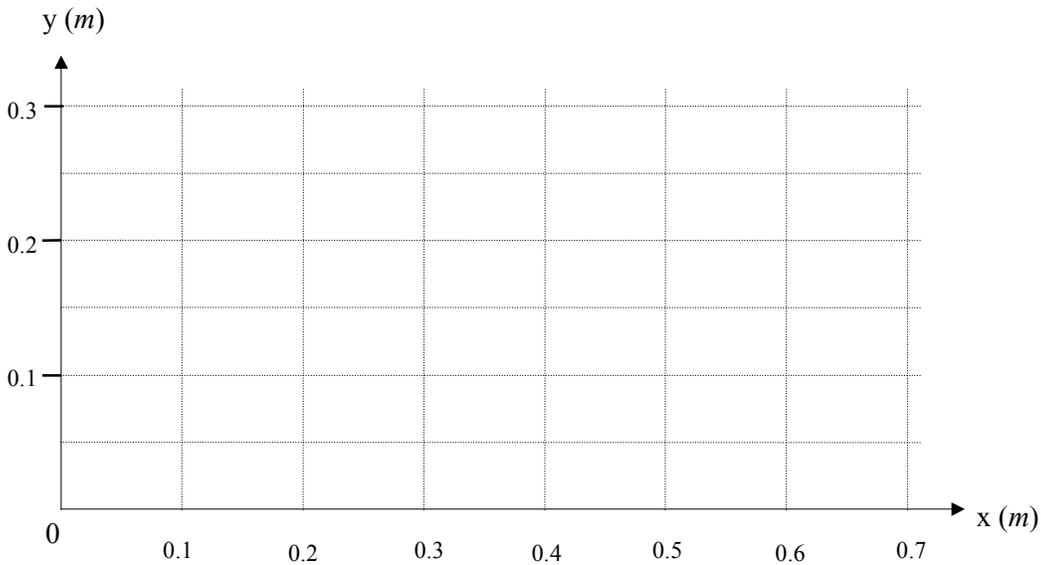
These initial parameters, which specify how you start the projectile *motion*, uniquely determine the shape of the projectile *path*.

Mapping Out the Parabola

First find the motion functions $x(t)$ and $y(t)$ that specify the position of the ball at any time t between the initial launch time $t = 0$ and the final hit-the-table time. *Hint:* Substitute the numerical values of your measured initial parameters y_0 and v_0 into *The Projectile Motion Equations*.

$$x(t) = \underline{\hspace{2cm}} t. \quad y(t) = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} t - \underline{4.9} t^2 .$$

Compute $x(t)$ at $t = 0, 0.05 s, 0.10 s, 0.15 s, 0.20 s$ and mark these five values of x by placing five dots at the appropriate locations on the $x(m)$ axis shown below. Compute $y(t)$ at the same five times and mark these five values of y on the $y(m)$ axis below.



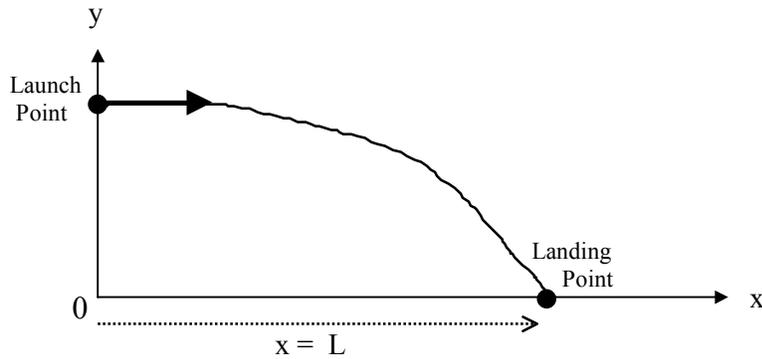
Place five dots within the xy plane pictured above that represent the actual five positions of the projectile at the five times considered above. Label each point with the corresponding time. Use these five points as a guide to draw the entire smooth path of the projectile as it flies through the air.

Find the equation $y(x)$ that describes this curved path. *Hint:* Eliminate t from your $x(t)$ and $y(t)$ equations by solving the x equation for t and then substituting this t expression into the y equation. You will be left with “ y as a function of x ” which should be a *quadratic function* of the form $y(x) = Ax^2 + C$. Show your derivation of $y(x)$ in the space below. The underlined coefficients that you fill in for the parabola equation $y(x)$ are the special “*parabolic parameters*” of your projectile motion.

$$y(x) = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} x^2 .$$

Where Does the Projectile Land ?

In theory, the landing point is defined by the coordinate point $(x, y) = (L, 0)$.



Use your *parabola equation* $y(x)$ to compute the horizontal landing distance L of your projectile. Show your calculation.

L (theory) = _____ m .

Launch the ball five times. Arrange for the ball to land on a piece of carbon paper, which is placed on top of copy paper taped to the table. The scatter of landing points (dots) recorded on the paper provides a nice visual display of the uncertainty in L . Find the average value of L and the uncertainty in L (half-width spread around the average).

L (m)					
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L (experiment) = _____ \pm _____ m .

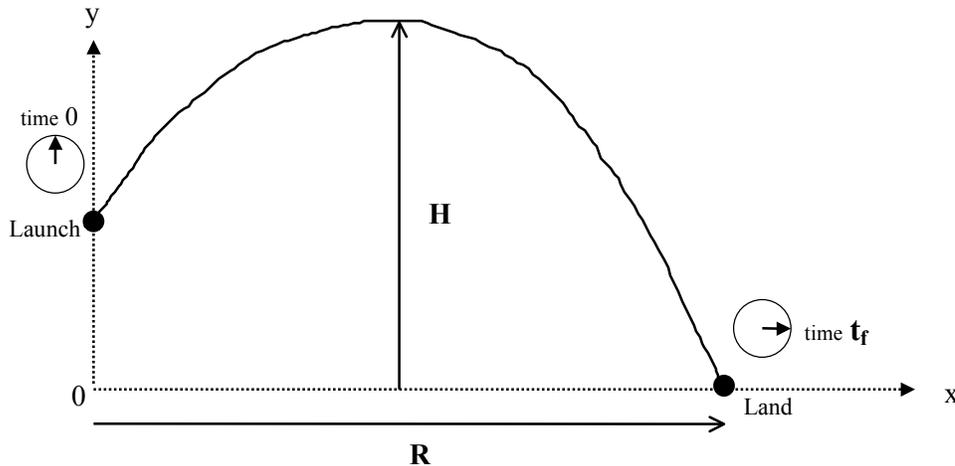
Does your value of L (theory) fall within the *range* of the values of L (experiment)?

% difference between L (theory) and L (experiment) is _____ % .

Part IV. Range, Altitude, Flight Time

There are three important properties of projectile motion:

- Range* $R \equiv$ Maximum horizontal distance.
- Maximum Height* $H \equiv$ Maximum vertical distance.
- Time of Flight* $t_f \equiv$ Time in air between launch and land.



Theory

The Motion Functions

Find the motion functions $x(t)$ and $y(t)$ that describe the motion of the ball projected at an angle of $\theta_0 = 60^\circ$ from your launcher. Use your measured values of y_0 and v_0 as the initial parameters in the motion equations. Remember: $v_{0x} = v_0 \cos \theta_0$, $v_{0y} = v_0 \sin \theta_0$. So as not to clutter the equations, do not include the units of the numbers that you write in the blanks below. But make sure that all your numbers are expressed in the metric units of *meters* and *seconds*.

$$x(t) = \underline{\hspace{2cm}} t. \quad y(t) = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} t - \underline{4.9} t^2.$$

The Parabola

Use your motion functions $x(t)$ and $y(t)$ to compute the position (x, y) of the ball every tenth of a second ($t = 0.0, 0.1, 0.2, 0.3, \dots$) until the ball hits the table. Note that the ball hits the table when $y = 0$. Record the position coordinates (x, y) in the data table below.

Position Coordinates (x, y) of the Projectile

t (s)	x (m)	y (m)
0.0		
0.1		
0.2		
0.3		
0.4		
0.5		
0.6		
0.7		

Note: Negative values of y denote the vertical position of the ball *below* the table surface. The ball does not have access to this region of space unless you cut a hole in the table for the ball to whiz through !

The x and y columns of your data table describe the parabolic path of the ball through the air. Use the program *Graphical Analysis* (on the Desktop) to graph y versus x. Include at least one negative value of y on your graph. Fit the points (x, y) on your graph with a *quadratic* (parabolic) function: $y = Ax^2 + Bx + C$. PRINT the graph showing the parabolic fit. Report the equation of your parabola:

$$y = \underline{\hspace{2cm}} x^2 + \underline{\hspace{2cm}} x + \underline{\hspace{2cm}} .$$

Finding R , H , t_f

Find the horizontal range R and the maximum height H of the parabolic path *directly on your printed graph*. DO NOT solve any equations. Simply look at your graph of the parabola. Draw the horizontal line representing the table surface (the x-axis) on your graph. Write “Table Surface” on this line. Mark the value of R on this x-axis. Mark the value of H on the y-axis. Report your marked values of R and H here. Note: You can only read numbers on the x-axis and y-axis to *two* significant figures.

$$R \text{ (theory)} = \underline{\hspace{2cm}} \text{ m.} \quad H \text{ (theory)} = \underline{\hspace{2cm}} \text{ m.}$$

Use your x(t) equation and value of R to calculate the time of flight t_f. Show your calculation.

$$t_f \text{ (theory)} = \underline{\hspace{2cm}} \text{ s .}$$

Experiment

Set the launcher for a $\theta_0 = 60^\circ$ launch. Launch the ball and measure t_f , H , and R . One person should measure t_f – the “hang time” of the ball through the air – with a stopwatch. Another person should measure H – the maximum *altitude* – with a meter stick and a simple visible inspection of where the trajectory of the ball peaks. Use carbon paper on top of copy paper to record the landing point. Measure the range R – the horizontal distance between the launch point and the landing point – with a meter stick.

Repeat this 60° launch four more times. Fill in the table. Find the average values of your measured parameters R , H , and t_f .

Experimental Values of the Projectile Motion Parameters

						Average
$R (exp)$						m
$H (exp)$						m
$t_f (exp)$						s

Compare Theory and Experiment

% diff between $R (theory) =$ _____ and $R (exp) =$ _____ is _____ % .

% diff between $H (theory) =$ _____ and $H (exp) =$ _____ is _____ % .

% diff between $t_f (theory) =$ _____ and $t_f (exp) =$ _____ is _____ % .

If any percent difference is greater than 10%, then consult your instructor.

Part V. Design Problem: Human Cannonball.

A circus performer is launched from a cannon and lands in a net. Your tabletop ball launcher represents a small-scale working model of this human cannonball system. Your goal is to figure out *what angle to launch the ball* and *where to place the net* so that the flight time of the ball is equal to *0.50 seconds* and the ball lands in the net.

The Theory

Write the motion equations $x(t)$ and $y(t)$ for this problem. Use your previously measured values of the “cannon” parameters, y_0 and v_0 , as the initial parameters in the equations.

$$x(t) = \text{_____} \cos\theta_0 t . \quad y(t) = \text{_____} + \text{_____} \sin\theta_0 t - \text{_____} t^2 .$$

Substitute the specified value of $t = t_f = 0.50 \text{ s}$ into $x(t)$ and $y(t)$. Remember: At the time $t = t_f$, the ball is at the landing point $(x, y) = (R, 0)$. Solve your two equations for the two unknowns θ_0 and R . Show all your algebra in the space below.

$$\theta_0 = \text{_____}^\circ . \quad R = \text{_____} \text{ m} .$$

Ask your instructor to visit your table with the “net” in order to check your theory and run the experiment with your team.

The Experiment

Set the launch angle of the cannon at your theoretical (predicted) value of θ_0 . Place the landing net on the table at your theoretical (predicted) value of R . Note that the *radius of the net* defines the allowed *uncertainty in R*. Launch the ball. Observe where the ball lands.

$$\text{Predicted } R = \text{_____} \text{ m} \text{ and } \text{Actual } R = \text{_____} \text{ m} \text{ differ by } \text{_____} \% .$$

Now launch the ball again and measure the time of flight.

$$\text{Specified } t_f = \underline{0.50} \text{ s} \text{ and } \text{Actual } t_f = \text{_____} \text{ s} \text{ differ by } \text{_____} \% .$$