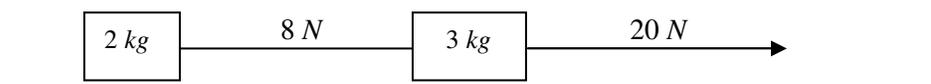


Force and Motion

In previous labs, you used a motion sensor to measure the position, velocity, and acceleration of moving objects. You were not concerned about the “mechanism” that caused the object to move, i.e. the forces that acted on the object. In other words, you answered the *kinematic* question “How do objects move?” Now it is time to answer the *dynamic* question “Why do objects move?” Force causes motion. The *force* of gravity causes the earth to *move* around the sun. The *force* of electricity causes the electron in an atom to *move* around the proton. The strong quantum force “glues” the protons (quarks) inside the tiny nucleus of an atom.

Newton’s celebrated equation of motion, $F = ma$, describes the precise connection between *force* (F) and *motion* (a) in classical mechanics. You have used a motion sensor (accelerometer) to measure a . You will now use a “force-ometer” to measure F . By measuring the left side of $F = ma$ with a force meter and the right side with a motion meter, you will discover the deep relation between F and ma firsthand in the laboratory.

Thought Experiment 1. A 2 kg mass and a 3 kg mass are connected by a massless cord and move on a horizontal frictionless surface. The 3 kg mass is pulled to the right with a force of 20 N. The tension in the cord is 8 N. Fill in the “ F ” and “ ma ” blanks.



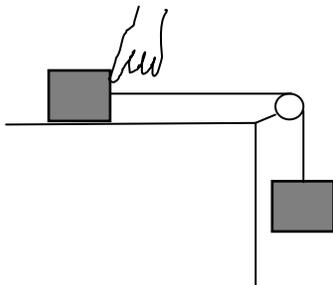
Force: Net Force on 2 kg mass = _____ N.

Net Force on 3 kg mass = _____ - _____ = _____ N.

Motion: Mass \times Accel. of 2 kg mass = _____ \times _____ = _____ $kg\ m/s^2$.

Mass \times Accel. of 3 kg mass = _____ \times _____ = _____ $kg\ m/s^2$.

Thought Experiment 2. Two identical masses are connected by a cord. One mass is on a horizontal surface and the other hangs over a pulley. Neglect friction. When the system is at rest (while holding the mass on the surface with your hand), the tension in the cord is 10 N. When the system is accelerating (after you remove your hand), what is the tension in the cord?



Part I. “Forceometers”

The Spring Scale

The spring scale is the prototype “forceometer”. For centuries, springs have been used to measure force. The force required to stretch a spring is proportional to the amount of stretch. A spring scale is a spring that has been calibrated to convert the amount of stretch (*meters*) into the corresponding value of the force (*Newtons*).

Examine the spring scale. Note that there is a large fixed hook at one end and a small hook at the other end. The small hook is attached to a sliding rod, which is connected to the spring. Pull on the small hook so that the spring is stretched several centimeters (half its maximum stretch) and the reading on the force scale is 0.5 N. Double the stretch of the spring so that the force is 1.0 N (maximum scale reading).

You already know what a “one pound force” feels like. But what does a “one Newton force” feel like? Pull on the spring scale until you have a good *kinesthetic sense* of a one Newton force.

Zero the spring scale as follows. Hold the spring scale in the vertical position with the small hook downward. With nothing attached to the hook, make sure the scale reads zero. If the reading is not “0”, then turn the screw at the top of the tube until zeroed. Once the scale is zeroed, hang a 50 *gram* mass from the small hook. What is the value of the force F recorded by the spring scale?

Spring Scale Reading: $F = \underline{\hspace{2cm}} N.$

Since the 50 *gram* mass is at rest, the upward pull of the spring on the mass is equal to the downward pull of the earth on the mass. The pull of the earth – the force of gravity – on an object is called the **Weight** of the object. Calculate the theoretical value of the weight of a 50 *gram* mass.

Calculated Weight of 50 *gram* mass: $W = \underline{\hspace{2cm}} N.$

% diff between *Measured Weight* (spring scale reading) and *Calculated Weight* is $\underline{\hspace{1cm}}$ %.

The Force Sensor

Examine the force sensor. The switch on the force sensor should be set to the 10 *N* range. The force sensor is an electronic strain gauge. Any force (push or pull) on the hook causes a metal block inside the sensor to bend slightly. The strain in the block is converted into an electrical signal (voltage) which is then converted, via the interface box, into the corresponding value of the mechanical force. So like the spring scale, the force sensor converts a distance (strain in *meters*) into a force (*Newtons*).

The force sensor measures $F(t)$ = Force on the hook as a function of time. To activate the force sensor or “force probe”, start *Logger Pro* and open the file *ForceProbe*. Click on *Collect*. Push and pull on the hook and note the digital values of $F(t)$ that are displayed in the data table and the corresponding $F(t)$ curve that is traced out in the graph window.

Zero the force sensor as follows. While holding the force sensor in the vertical position with the hook downward and nothing attached to the hook, click on the *Zero* button (to the right of the *Collect* button). Once the sensor is zeroed, hang a 50 gram mass from the hook and click on *Collect*. Average the $F(t)$ data. What is the value of the force F recorded by the force sensor ?

Force Sensor Reading: $F =$ _____ N .

% diff between *Measured Weight* (force sensor reading) and *Calculated Weight* is _____ %.

Part II. Apply F . Measure ma . Prove the Law $F = ma$.

Measure F with the Spring Scale

Place the 500-gram cylinder weight on top of the cart. The spring scale will be used to apply a force F to the cart. Make sure that the spring scale reads “zero” when it is held in the *horizontal* position and nothing is attached to the small hook. Attach the small hook of the spring scale to the cart. Place the cart on the table surface – not on the track.

CAUTION: The cart loaded with the weight is a heavy object. DO NOT let the cart roll off the table. It could injure your foot.

Pull the cart along the table in a straight line with a force of 0.15 N . Remember to keep the spring-scale tube horizontal (do not tilt the tube) while you are pulling. It may be a bit tricky to keep the force constant. In experimental physics, nothing is exactly constant. It is natural for the force to fluctuate around 0.15 N. Sometimes you pull too hard, sometimes you pull too soft, but on average you pull just right. Practice pulling the cart so that you are able to keep the force-scale reading confined to the region between 0.10 N and 0.20 N. The average force will then be 0.15 N. The experimental value of the force can be reported as

$$F = \underline{0.15 N} \pm \underline{0.05 N} .$$

Note that this experimental uncertainty is larger than 10%. If your team can consistently pull with a steadier force, say $0.15 N \pm 0.03 N$, then by all means report your more accurate measured value of F in the space above and use it in the following analysis.

Measure ma with a Stopwatch

To determine the acceleration a of your cart, you will measure *time* and *space*. Use a *stopwatch* to measure the time t it takes for your cart – *starting from rest* and pulled with the force $F = 0.15 N \pm 0.05 N$ – to cover the distance $d = 1.0$ m along the table.

To measure d and t accurately, lay a meter stick on the table. The stick is the “x axis”. Pull the cart alongside (but not touching) the meter stick. Keep the cart’s line of motion parallel to the stick.

Use the kinematic relation $d = \frac{1}{2} a t^2$ to compute a from your measured values of t and d . Repeat the run five times. Record your five measured values of t in the *motion table* below. For each run, compute the values of a and ma .

Measure the total mass of your cart (cart + bar) using the mass scale: $m = \underline{\hspace{2cm}}$ kg.

Motion Table

d (m)	t (s)	a (m/s^2)	ma (N)
1.0			
1.0			
1.0			
1.0			
1.0			

Compute the average value of ma . Estimate the uncertainty from the half-width spread in your five values of ma : $Uncertainty = (ma_{max} - ma_{min}) / 2$.

$$ma = \underline{\hspace{2cm}} \pm \underline{\hspace{2cm}} N.$$

Compare ma and F

You have measured the *force* quantity F using a *spring scale*. In a completely independent measurement, you have measured the *motion* quantity ma using a *stopwatch* (and meter stick and mass scale). Here is the BIG question: Is F equal to ma ?

Do your measured values of F and ma provide an experimental proof Newton’s famous second law of motion? Justify your answer in the space below.

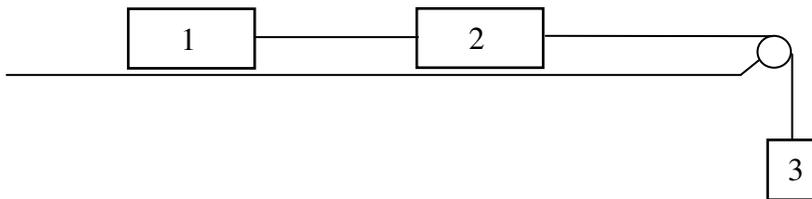
Remember: since an experimental value ($number \pm uncertainty$) is really a *range* of numbers, “comparing two values” really means “comparing two ranges”.

Part III. A Three-Body Problem

Do this experiment with the team across the table (two computers are needed). In the previous part, you studied a system consisting of only one mass (body). In physics, the “*two-body problem*” refers to any dynamical problem involving two interacting objects. A famous “*three-body problem*” is solving the equations of motion for the Sun-Earth-Moon system.

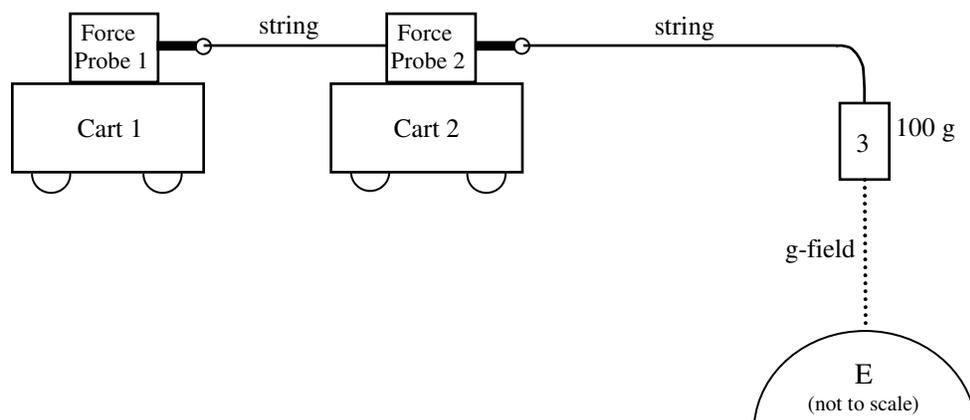
In this part, you will study a mechanical system composed of three connected masses. You will discover how the *net force* – the sum of all the individual forces – acting on a body is the crucial dynamical quantity that determines the motion of the body. *May the net force be with you!*

Here is a schematic of your three coupled bodies (1 and 2 on a horizontal track and 3 hanging over a pulley):



Strictly speaking, this is a 4-body problem. The Earth (E) is the fourth body connected to 1, 2 and 3 via the gravitational field. Note that E only affects the vertical motion of 3.

Build the Three-Body System. Body 1 consists of cart 1 + force probe 1. Body 2 consists of cart 2 + force probe 2. Each force probe should be mounted on top of the cart via a bracket. Body 3 is a 100-gram cylinder weight. Use a string for the 1-2 coupling and another string for the 2-3 coupling. Nature has automatically installed the “string” between 3 and E – it is called the “*g-field*”.



Note that the coupling (string tension) between 1 and 2 is measured by force probe 1, while the coupling (string tension) between 2 and 3 is measured by force probe 2. The coupling between 3 and E is mg , where m is the mass of 3 and g is the gravitational field due to E.

Measure the mass of Body 1 (cart + probe) and the mass of Body 2 (cart + probe).

$$m_1 = \underline{\hspace{2cm}} \text{ kg} \quad m_2 = \underline{\hspace{2cm}} \text{ kg} \quad m_3 = \underline{0.100} \text{ kg}$$

Measure the Acceleration

First make sure your track is level using the “steel ball test”. Place Body 1 and Body 2 on the track and hang Body 3 over the pulley. Tilt the pulley so that string is horizontal and does not rub on the bracket. Start the system so that the bottom of Body 3 is at a height of 60 cm above the ground.

The same person should release the system and measure the time as follows: Hold Body 1 (the rear cart) with your hand to keep the system from moving. Release your hold and use a stop watch to measure the time t it takes for Body 3 to hit the ground. Repeat two more times. Find the average time and compute the acceleration a of the system in the space below:

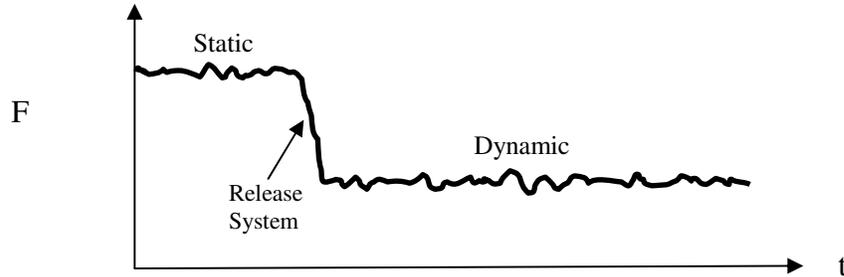
$$t = \underline{\hspace{1cm}} , \underline{\hspace{1cm}} , \underline{\hspace{1cm}}$$

$$a = \underline{\hspace{2cm}} \text{ m/s}^2.$$

Measure the Forces

1. Open the file *Force Probe* on both computers.
2. Change the time scale to go from 0 to 3.0 s on both F vs t graphs.
3. Change the force scale for Force Probe 1 to go from 0.0 N to 1.1 N (computer connected to rear cart). Change the force scale for Force Probe 2 to go from 0.70 N to 1.2 N (computer connected to front cart).
4. Zero each force probe while the strings are slack (no force must be pulling on the force probes tips).
5. Start with the hanging mass above the ground and hold Body 1 (rear cart) to keep the system initially at rest.
6. Make sure the hanging mass is NOT SWINGING. It must be completely at rest. This minimizes oscillations in the string and force graphs.
7. Click Collect (hit space bar) on both computers at the same time. DO NOT let go of the cart yet. WAIT until you see the static force value (horizontal line at about 1.0 Newton) appear on graph 1. Once this line appears, release your hold.

Look at your F vs t graphs. Note how the force changes (drops in value) from a **static** value when the system is at *rest* (before you release the hanging mass) to a **dynamic** value when the system is *accelerating* (after the release):



Find the average dynamic force measured by Force Probe 1 and the average dynamic force measured by Force Probe 2. Remember to first highlight the good data region (flat dynamic line) before you hit the STATS button. Record the force probe readings in the space below. Run the experiment again and measure the forces.

Trial 1 Force Probe 1 = _____ N . Force Probe 2 = _____ N .

Trial 2 Force Probe 1 = _____ N . Force Probe 2 = _____ N .

Record the average value of your two trials for each force probe. Also record the gravitational force on body 3.

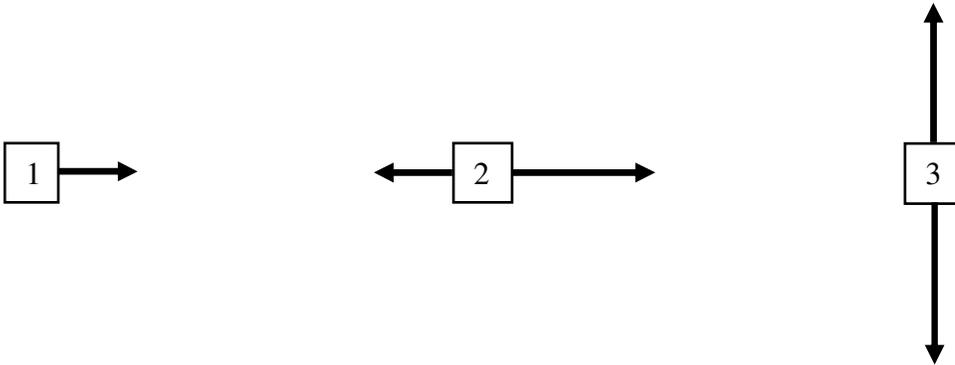
Force Probe 1 = _____ N . Force Probe 2 = _____ N .

Gravitational Force on Body 3 = _____ N .

Free-Body Diagrams

Write your three force values (recorded above) next to the corresponding force vectors on the following free-body diagrams. These diagrams only show the forces in the direction of motion. Neglect friction.

Free-Body Diagrams



Net Force on Each Body

Use your force diagrams to find F_{net} for each body.

$$F_{1net} = \text{_____ N} \quad F_{2net} = \text{_____ N} \quad F_{3net} = \text{_____ N}$$

Mass \times Acceleration of Each Body

Use your measured values of masses and acceleration to find ma of each body.

$$m_1 \times a_1 = \text{_____} \times \text{_____} = \text{_____}$$

$$m_2 \times a_2 = \text{_____} \times \text{_____} = \text{_____}$$

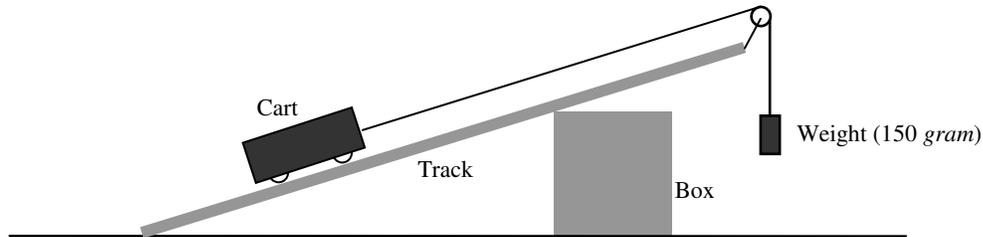
$$m_3 \times a_3 = \text{_____} \times \text{_____} = \text{_____}$$

Newton's Law

Newton's *second law of motion* says that the net F on a given body equals ma of that body. For each body (1, 2, 3) in your experiment, how well do your measured values of F and ma provide an experimental proof of this celebrated law of nature?

Part IV. Creating a Physics Sculpture

Title: “Motionless Bodies on a Tilted Beam”

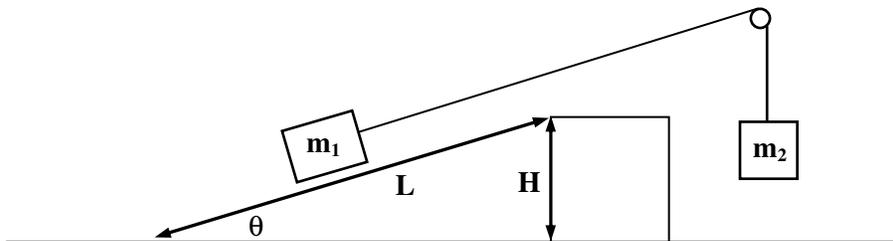


Depending on the tilt of the track, the cart will accelerate up or down. However, there exists one special tilt angle such that the two-mass system is in “*Equilibrium*” – a special state where the masses do not move at all! In this state of *zero acceleration*, the net force on each mass is equal to zero. In the sculpture above, the forces of gravity and tension are in *perfect balance*.

Design Goal: Where should you place the box so that the masses are motionless? Work out the theory behind this mechanical structure *before* you actually build the structure.

The Theory

Here is a schematic of the system showing the *system parameters* m_1 , m_2 , L , H .



Answer the following questions using only symbols. No numbers allowed.

1. Draw two free body diagrams, one for m_1 and one for m_2 .



2. Set up Newton’s equations: $F_{\text{net } 1} = m_1 a$ for body 1 and $F_{\text{net } 2} = m_2 a$ for body 2.

3. Solve Newton's equations to find L as a *function* of m_1 , m_2 , and H . Hint: Express $\sin\theta$ in terms of L and H .

$$L = \underline{\hspace{2cm}} .$$

Measure the values of H and m_1 . The value of m_1 is the mass of the cart alone, so make sure the force sensor is no longer attached to the cart. The value of m_2 is given to be 150 *grams*.

$$H = \underline{\hspace{2cm}} \text{ m} . \quad m_1 = \underline{\hspace{2cm}} \text{ kg} . \quad m_2 = \underline{0.150} \text{ kg} .$$

Plug the values of these system parameters into your theoretical formula for L .

$$L \text{ (theory)} = \underline{\hspace{2cm}} \text{ m} .$$

Note that L is the key architectural element in the design because it is the system variable that determines the overall geometry (tilt) of the sculpture.

Building the Sculpture

Use your theoretical value of L to build the actual sculpture.

Construction Hints: Remember to measure L from the point where the *track touches the table* to the point where the *track touches the box*. In your theory, you neglected friction and assumed tension \mathbf{T} was parallel to the track. Therefore, in your sculpture, you must make sure that the string does not rub on the bracket and that the string is parallel to the track.

Did your team achieve “perfect balance” the moment you placed the box at $L \text{ (theory)}$? Explain. Compare $L \text{ (actual)}$ with $L \text{ (theory)}$.