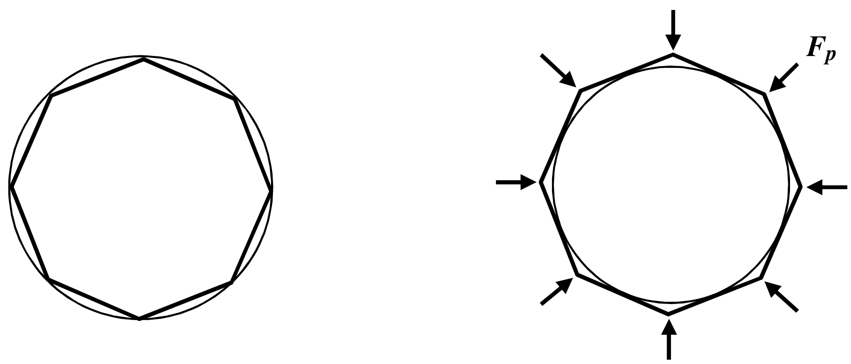
Circular Motion

I. Centripetal Impulse

The "centripetal impulse" was Sir Isaac Newton's favorite force.

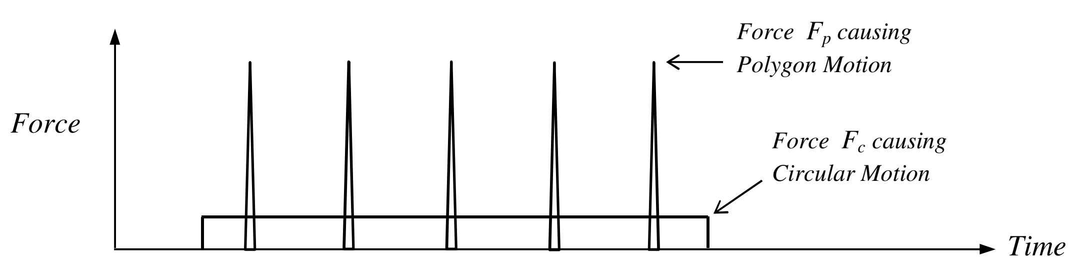
The Polygon Approximation. Newton made a business of analyzing the motion of bodies in circular orbits, or on any curved path, as motion on a polygon. Straight lines are easier to handle than circular arcs. The following pictures show how an inscribed or circumscribed polygon approximates a circular path. The approximation gets better as the number of sides of the polygon increases.



To keep a body moving along a circular path at constant speed, a force of constant magnitude that is always directed toward the center of the circle must be applied to the body at all times. To keep the body moving on a polygon at constant speed, a sequence of *impulses* (quick hits), each directed toward the center, must be applied to the body only at those points on the path where there is a bend in the straight-line motion.

The force F_c causing uniform circular motion and the force F_p causing uniform polygonal motion are both centripetal forces: *"center seeking" forces that change the direction of the velocity but not its magnitude.*

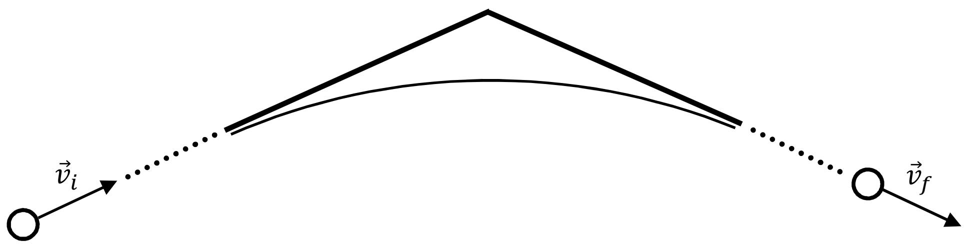
A graph of the magnitudes of F_c and F_p as a function of time would look as follows:



In essence, F_p is the *digital version* of F_c. A polygon is the *digital version* of a circle.

Applying a Centripetal Impulse

Here you will get a kinesthetic *sense* of Newton's favorite force. Your team will learn what it "feels like" to apply the force F_p that causes a ball to move on a polygon. The bent-line polygonal path and the circular arc it approximates appear on a sheet of paper at your table.



DO NOT tape this sheet to the table. DO NOT write on the sheet. DO NOT let ball roll off table.

For a ball moving at constant speed along the left line in the picture, how do you get the ball to suddenly turn the corner or "round the bend" and end up moving at the same constant speed on the right line?

1. Place one block under the end of the ramp. Start the ball at the top of the ramp. Let the ball roll down the ramp onto the dashed line (initial velocity vector) that leads into the polygon segment (bold straight line).

2. When the ball reaches the bend, use the <u>rubber tip</u> of the force probe to apply one "quick hit" to the ball so that the ball successfully rounds the bend – but does not noticeably change its speed. The direction of your push is crucial – this special direction is a huge part of the physics of circular motion! Do not record the force. Your goal here is simply to experience first-hand a "centripetal nudge".

3. Practice the push several times until you get a feel for the <u>*magnitude*</u>, the <u>*duration*</u>, and the <u>*direction*</u> of the force $\vec{F}_p$. All team members should give it a try.

In the space below, sketch a picture that shows the path of the ball, the force vector $\vec{F}_p$ acting on the ball, and the acceleration vector $\vec{a}$ of the ball. Note that $\vec{a}$ is proportional to $\vec{v}_f - \vec{v}_i$. Estimate the magnitude (average value) and the duration (how long it lasts) of your push – this is a rough estimate since it is based solely on what you felt while pushing and on your intuition about *force* and *time*.

Magnitude of $\vec{F}_p \approx$ _____ N. Direction of $\vec{F}_p$ is ___________. Duration of $\vec{F}_p \approx$ _____ s.

Before you move on to the next part, ask your lab instructor to check your pushing technique

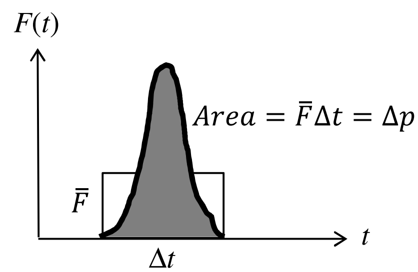
II. Scattering Experiment. Impulse and Momentum.

Impulse-Momentum Theorem. Newton's law of motion can be written as $\vec{F}dt = md\vec{v}$ or

$$\int \vec{F}dt = m\vec{v}_f - m\vec{v}_i \ .$$

This relation says $\vec{J} = \Delta\vec{p}$, where *impulse* is $\vec{J} \equiv \int \vec{F}dt$ and *momentum* is $\vec{p} \equiv m\vec{v}$. Impulse measures both the *strength* and the *duration* of the force. Note that $\int Fdt = \bar{F}\,\Delta t$, where $\bar{F}$ is the average of F over the time interval Δt during which F acts. The impulse-momentum theorem reveals a wonderful relation between AREA (*force* × *time*) and CHANGE (*change in momentum*):

Area under F(t) Curve = Change in Momentum

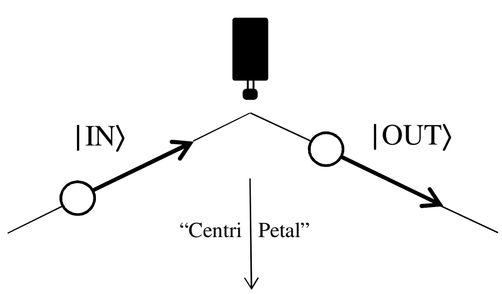


Measuring the Centripetal Impulse J_p

1. Open Logger Pro file *"Force Probe"*. Go to Experiment, click on Data Collection, and set the *Time Length* to <u>3 seconds</u> and the *Sampling Rate* to <u>1000 samples per second</u>. ZERO the force probe while it is in the horizontal position.

2. As before, start the ball at the top of the ramp. Let the ball roll down the ramp and onto the dashed line that leads into the polygon segment.

3. Instead of moving the probe to hit the ball, keep the probe STATIONARY – *pointed in the centripetal direction* – and let the ball hit the rubber tip of the probe at a glancing angle. During this **scattering process**, the ball makes a **transition** from the **in state** $\vec{v}_i$ to the **out state** $\vec{v}_f$. This scattering technique is more reproducible than "hitting" the ball. Place the force probe on its side (small face on the table) – this should make the rubber tip (line of force) even with the center of the ball. Push down on the probe so that the probe does not move when hit by the ball. You will have to run some practice trials, fine-tuning the probe position each time, so that the incoming ball scatters off the probe, turns the corner, and ends up moving on the outward polygon line.



4. Change the scales on the *Force* vs *Time* graph so that the shape (height and width) of the "triangular force spike" fills most of the graph window. The magnified force curve should resemble a "bell curve".

5. Compute the impulse J_p of your force-probe hit F_p by highlighting the force curve on you graph and clicking on the area icon . Record the value of J_p in the table below.

6. Find the average force $\bar{F}_p$ (use the STATS button). Be careful to only highlight the force curve – DO NOT highlight any region where the force is zero. Note that $\bar{F}_p$ is about one half the maximum force (height of "force triangle"). Find the duration time Δt_p (width of "force triangle") using the examine icon or viewing the bottom left corner of the graph window. Enter the values of $\bar{F}_p$ and Δt_p in the table. Their product should be equal to J_p within experimental error.

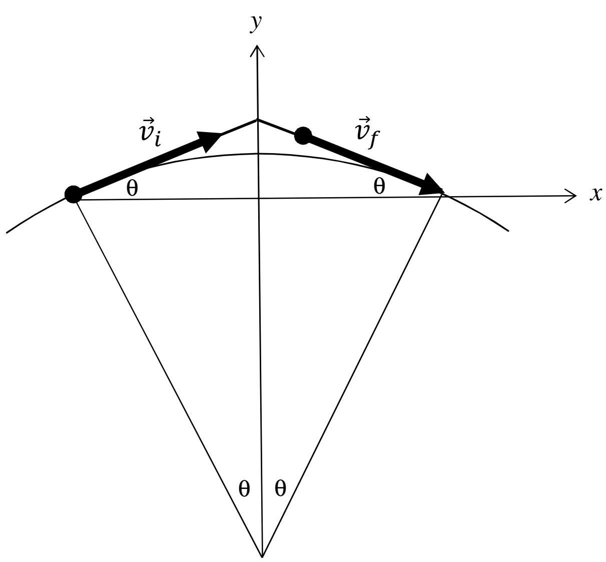
7. Do three more trials. Average your J_p values. Find the uncertainty (half-width spread).

Centripetal Impulse J_p				
Average Force $\bar{F}_p$				
Duration Time Δt_p				

J_p = ______________ ± _____________ Ns .

Measuring the Momentum Change Δp

The velocity vectors of the ball along the two polygon lines are drawn below. Note that $\vec{v}_i$ is also the velocity of the ball when it enters the circular arc while $\vec{v}_f$ is the velocity when it exits. Since the speed of the ball is constant, the *magnitudes* of $\vec{v}_i$ and $\vec{v}_f$ are the *same* but their *directions* are *different*.



1. Use this velocity diagram to calculate the change in the velocity vector: $\Delta\vec{v} \equiv \vec{v}_f - \vec{v}_i$.

Hint: Express each velocity vector in terms of its *x* and *y* components and then subtract the vectors. Show your vector algebra next to the diagram above. Express your answer in terms of the variables *v* and θ, where *v* is the constant speed of the ball, $v = |\vec{v}_i| = |\vec{v}_f|$, and θ is the angle defined in the diagram.

$$\vec{v}_f - \vec{v}_i = \boxed{}$$

2. How is the direction of $\Delta\vec{v}$ related to the direction of $\vec{F}_p$? ______________________________ .

3. Measure the mass *m* of the ball. m = _________ kg .

4. Measure the angle θ. Hint: Use a protractor to measure the angle (2θ) subtended by your circular arc (see diagram). θ = __________ .

5. Measure the speed *v* of the ball as follows. Start the ball at the same place as before (top of ramp). Place the small ruler at the end of the ramp so that when the ball rolls off the ramp onto the table, the ball moves alongside the ruler. Measure the time it takes the ball to cover the length of the ruler. Different team members should make this measurement to make sure you are getting a consistent value. Display your data and calculation:

v = __________ m/s .

6. Compute the momentum change $\Delta\vec{p} \equiv m\,\Delta\vec{v}$ in the space below:

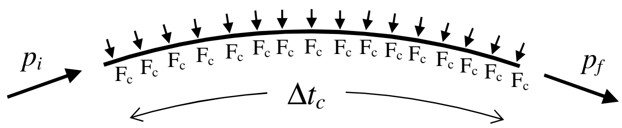
$\Delta\vec{p}$ = __________ kg·m/s .

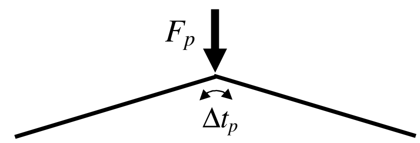
Does your Experimental Data "Prove" the Theorem $\vec{J}_p = \Delta\vec{p}$?

You have measured $\vec{J}_p$ with a force probe. You have measured $\Delta\vec{p}$ with a ruler, protractor, stopwatch, and mass scale. Compare the directions of $\vec{J}_p$ and $\Delta\vec{p}$. Compare their magnitudes. Does Δp fall within the *uncertainty range* of J_p ?

Centripetal Force F_c

The centripetal force $\vec{F}_c$ acts continuously over the whole circular arc, while the polygon force $\vec{F}_p$ acts momentarily only at the bend in the polygon.





$$\bar{F}_c \, \Delta t_c \;=\; \Delta p \qquad\qquad \bar{F}_p \Delta t_p \;=\; \Delta p$$

The relation $\bar{F}_c \, \Delta t_c = \Delta p = \bar{F}_p \, \Delta t_p$ says that a small force $\bar{F}_c$ acting over a long time Δt_c causes the same momentum change Δp as a large force $\bar{F}_p$ acting over a short time Δt_p. Hence, a formula for the centripetal force is

$$F_c \;=\; \frac{\Delta p}{\Delta t_c} \,.$$

Comment: We have set the average of the force equal to the magnitude of the force: $\bar{F}_c = F_c$. The exact relation is $\bar{F}_c = F_c \frac{sin\theta}{\theta}$, but the angular factor is near unity for the short arcs in this lab.

Use this formula to calculate the value of the centripetal force F_c in your experiment. You have already measured Δp for your circular arc. You can find the time Δt_c it takes the ball to transit the circular arc from your measured value of the speed v of the ball. All you need to do is measure the length ΔL_c of your circular arc using a piece of string: ΔL_c = ________ *m*.

$$F_c \;=\; \frac{\Delta \mathrm{p}}{\Delta t_c} \;=\; \frac{\rule{3cm}{0.4pt}\; Ns}{s} \;=\; \rule{2.5cm}{0.4pt}\; N\,.$$

The most famous formula for the centripetal force F_c (which readily follows from $F_c = \Delta p/\Delta t_c$) is

$$F_c \;=\; \frac{mv^2}{R}.$$

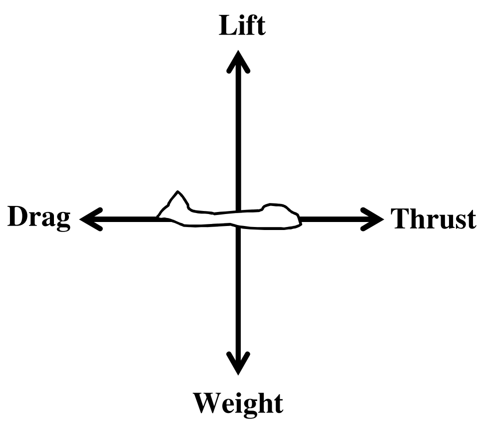
Use this formula and your known values of m, v, and F_c to calculate the *theoretical* value of the radius R of your circular arc. Measure the *actual* value of R with a ruler. Compare to your theoretical value.

III. Aerodynamics: Airplane Mass and Gee Force

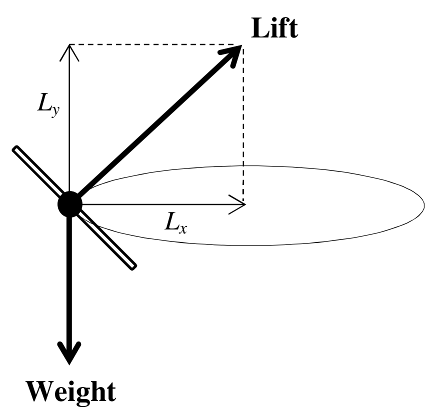
The four forces of flight are *Lift*, *Weight*, *Thrust*, and *Drag*. *Lift* and *Weight* control the "up and down" motion of the plane (or bird) while *Thrust* and *Drag* control the "fast and slow" motion. *Lift* is due to the air (and the wing shape), *Weight* is due to the earth, *Thrust* is due to the propeller or jet engine, and *Drag* is due to the air.

For a plane moving in a straight horizontal line at constant speed, *Lift* = *Weight* and *Thrust* = *Drag*.
For a plane moving in a circular horizontal path at constant speed, *Lift* > *Weight* and *Thrust* = *Drag*.
Note that the circular motion is a type of "banked" motion (the wings are tilted) where the horizontal component L_x of the *Lift* provides the centripetal force. The vertical component L_y balances the weight.

Linear Motion

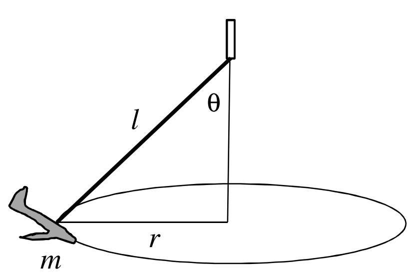


Circular Motion

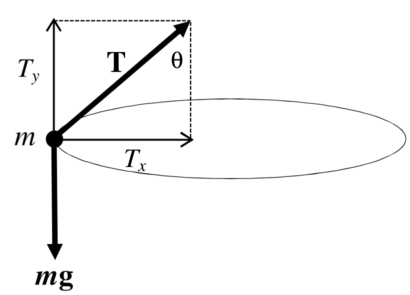


In today's experiment, your goal is to find the mass of a plane (without weighing it) and the "*g*-force" on the pilot by observing the motion. The model system consists of a toy airplane suspended from a cord that travels in a horizontal circle.

"Real World"



"Force World"



Note: the "lift force" on the toy plane is provided by the tension **T** in the cord.

The Theory

In the space below, carefully derive the formula that gives the mass *m* of the plane as a function of three system parameters: l = length of cord, T = tension in cord, and τ = period of motion.

Hints: Look at the free-body diagram above. The centripetal force $T_x = T \sin\theta$ causes the plane to move in a circle with a centripetal acceleration $a = v^2/r$. The speed v of the plane is distance ($2\pi r$) over time (τ). Express $\sin\theta$ in terms of r and l.

m = ________________________ .

Have your instructor check your theoretical work before you move on to the experimental phase

The Experiment

Read this paragraph *before* you turn on the plane's propeller. To calculate *m* using your theoretical formula, you need to measure T, l, and τ. The spring scale will read the tension T in the cord when the plane is in flight. The cord length l is the distance from the bottom of the plastic tube where the cord pivots to the "center of mass" of the plane – when the plane is in flight! To measure this in-flight l, you will have to stop the plane after the run and pull on the cord until the spring scale reads the in-flight tension. Use the meter stick to measure this l. Use the stopwatch to determine the period τ of the plane's motion. For accuracy, measure the time it takes the plane to complete ten revolutions and then divide by ten to obtain the period τ.

Turn on the plane's propeller. **CAUTION**: Rotating Propeller Blade!

Give the plane a push to launch it into a circular path. Allow the plane to circle several times so that its motion can stabilize before you take any measurements – but don't wait too long because the string becomes twisted (shortens) and the tension tends to increase. Two people should measure the time τ. The other two people should monitor the spring scale reading T during the ten revolutions. If the tension increased from 2 N to 3 N, then record the tension as 2.5 N.

Also use **CAUTION** when stopping the plane – stay away from the propeller – DO NOT use your hands. Ask your instructor for safe stopping techniques.

Record your measured values of the length *l*, force *T*, and time τ:

String length l (m)	*Cord Tension* T (N)	Time for Ten Revolutions	*Orbit Period* τ (s)

In the space below, calculate the mass *m* of your plane from your measured values of *l*, *T*, and τ.

Calculated m = ______________ kg.

Take your plane off the cord and weigh it on the scale at the back of the room.

Measured m = ______________ kg .

Compare *Calculated* m and *Measured* m.

g-force

Find the centripetal acceleration *a* of the plane in <u>units of g</u>. For example, if $a = 19.6$ m/s^2, then $a = 2g$ and hence the pilot experiences *two gees* of "force". Hints: Look at the free-body diagram. Note that T_y is equal to $T\cos\theta$. Use $T_y = mg$ and your measured values of *T* and *m* to compute θ. Combine $T_x = ma$ and $T_y = mg$ to get the elegant relation, $a = g \tan\theta$. Show all your work.

a = ___________ g.