Circular Motion

Names: ________________________
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Introduction

Many rides in carnivals or amusement parks give their customers thrills by swinging them in circles suspended by a long chain. Sometimes the rider is in a seat, sometimes standing, sometimes in an “airplane.” How does the force on the cable holding the rider depend on the length of the cable and the speed with which the rider goes around? In this laboratory you’ll explore this question with a toy airplane. We will also analyze a video clip of a toy plane in motion and determine its velocity and acceleration.

Part I: Weighing an airplane without using a scale

We will determine the mass of a toy airplane suspended from a cord. Assume the plane is traveling in a circle about the central pole.

1. Write down an expression for the velocity of the plane when it is orbiting the central pole in a circular orbit. Express your answer in terms of \( R \) (the radius of the plane’s orbit) and \( P \) (the period of the plane’s orbit).

2. Based on your answer above, what is the plane’s acceleration when it is in orbit? Express your answer in terms of \( R \) and \( P \).

3. In a free body diagram of the plane, you would use a dot to represent the plane. Indicate where this dot is located on the photo of the plane. What does the dot represent?
4. Draw a free body diagram of the plane when it is in flight. Determine the direction and magnitude of the net force acting on the plane in terms of $R$, $T$ (the tension in the cord when the plane is in flight), and $L$ (the distance from the bottom of the plastic tube where the cord pivots to the center of mass, when the plane is in flight).

5. Based on your answers to questions 2 and 4 above, write Newton’s Second Law for the airplane in flight. Use $m$ to represent the mass of the plane.

6. Find the mass of the plane in terms of $T$, $L$, and $P$.

Check your result with your instructor before continuing. Instructor’s Initials: 

7. The plane is suspended by a cord from a spring scale. The spring scale will read the tension in the cord when the plane is in flight.

$L$ can be determined by recording the tension mid-flight and then, after stopping the plane, pulling on the cord until the spring scale has the same reading. You can then measure the length of the cord. Consider your answer to question 3 when determining $L$.

To determine the period of the plane you may use Xinkro, a stopwatch program on your computer. Record the time for at least ten orbits. You may wish to do more orbits if you are concerned about your accuracy in counting.

Turn on the plane’s propeller and give it a push to launch it into a circular path. Allow the plane to circle several times before taking your measurements, so that its motion can stabilize.

You will make three separate determinations of the mass of the plane, at three different velocities. Change the velocity of the plane by changing the length of the cord. You can do this by standing on the table and moving the scale up or down the pole. Please be careful when climbing on and off the table. DO NOT stand on one of the stools! You may also attach a card to the plane’s tail to increase the drag, which will also change the plane’s velocity.
Fill in the following table:

<table>
<thead>
<tr>
<th>Trial</th>
<th>$T$ (N)</th>
<th>$L$ (m)</th>
<th>Additional Drag? (yes or no)</th>
<th>Number of revolutions</th>
<th>Time for number of revolutions (seconds)</th>
<th>$P$ (seconds)</th>
<th>Calculated mass of plane (kg)</th>
</tr>
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<tbody>
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</tbody>
</table>

8. Your calculated masses should agree to within 10%. If not, consult your instructor. Find the average of your three mass determinations.

Average mass = ______________ kg

9. Take your plane off the cord and weigh it on the scale at the back of the room.

Mass from scale = ______________ kg

10. Compare the average mass to the scale reading.

Percent difference of average mass from scale reading = ______________%

11. Think about the assumption(s) you made when you derived the formula for mass. List the major assumption(s). Do you think these assumptions were valid? If so, why? If not, what would you have to change to improve your results?

12. When the plane is at rest, hanging vertically, what does the spring scale read? What does this reading represent? Does it have the value you expect based on your results above?
Part II: Position, Velocity and Acceleration in Circular Motion

A. The Plane's position

In this part you will view the circular flight of a toy airplane and analyze its motion using stop-action techniques.

On your computer click on the icon to start the program World in Motion.
Click on the Open Video File button and select Plane.avi.
Click on the Play button and watch the plane fly in a circle.

Note: The video was taken with the camera at ground level and the plane suspended about 4 m above the camera from a point about 2 cm in from the end of a wooden rod. Evidence that the plane exerted a force on the rod is seen by the swaying of the rod.

Your task is to record the position of the plane as it moves twice around in a circle. First, set the origin of the coordinate system. The program normally sets the origin at the lower-left hand corner of the screen, but it is better to make the origin the point from which the plane is suspended.

To change the origin, click on Video Scale and select New Origin. Follow the directions given.

Now, mark the location of the plane in each frame. Here's how:
Move the mouse to place the cursor at an easily identifiable spot on the body of the plane.
Click the left mouse button to mark the location with a red dot.
Then click the right mouse button to advance the video to the next frame.
Continue marking locations until the video will not advance any farther (frame 30).

Note: If you make a mistake and want to change the location of a dot, you can correct it by stepping forward or backward to the frame you want to edit, place the cursor over the new position, and press the left mouse button. If you want to start over, open the video file and select plane.avi again.

Create a graph of the motion.
Click on Graphs and choose Option 1.
Click on Next. Tell the program that the plane moves in the $x$- and $y$- directions, and $y$ is horizontal.
Click on Next. Plot only position versus time.
Click on Next. The program will produce a graph of the $x$- and $y$-coordinates of the motion as a function of time.
Click on Steps to change the video frame and see where the plane is on the graph.

Use the graphs to answer the following questions:

1. At what four times is the $x$-coordinate of the plane's position equal to zero?

2. At what four times is the $y$-coordinate of the plane’s position equal to zero?
B. The plane’s period

1. What is the period of the motion? That is, how long does the plane take to make one complete orbit?

   \[ P = \underline{\text{____}} \text{s.} \]

Note: the period is not at exact integral number of frames.

2. Explain how you determined the period \( P \).

C. The plane’s orbital radius

Your next task is to measure the radius of the orbit. A good way is to make a graph of the \( y \)-component versus the \( x \)-component like the one below.

Here is how:
Create a custom graph by clicking on Graph>> until the table of measurements appears. Click on the grey bar labeled Click Here to Create a Custom Plot, then click first on the \( Y1(m) \) column, then on the \( X1(m) \) column. In a few seconds you’ll see a graph like the one above.
Select Graph Tools, then Horizontal Scale if you want to change the range displayed or to add vertical lines as was done here.
If you click on Steps you can see how the points on the graph represent plane locations on the video.

1. On the graph above mark the plane’s position at frames 3 and 5.
Find the \( x \)- and \( y \)-coordinates for the two positions by carefully placing the cursor over the mark on the graphs. You can then read the coordinates in the upper-right hand corner.

2. Calculate the magnitude of the radius vector and the angle, \( \theta \), the vector makes with the **positive** \( x \)-axis for frame 3 and frame 5.

\[
\begin{align*}
x_3 &= \underline{\text{_____}} \text{ m} \quad y_3 = \underline{\text{_____}} \text{ m} \quad R_3 = \underline{\text{_____}} \text{ m} \quad \theta_3 = \underline{\text{_____}} ^\circ \\
x_5 &= \underline{\text{_____}} \text{ m} \quad y_5 = \underline{\text{_____}} \text{ m} \quad R_5 = \underline{\text{_____}} \text{ m} \quad \theta_5 = \underline{\text{_____}} ^\circ
\end{align*}
\]

3. Find the average value \( R \) at frames 3 and 5. \( R_{\text{average}} = \underline{\text{_____}} \text{ m} \)

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D. **The plane’s velocity**

Velocity is defined as \( \mathbf{v} = \frac{d\mathbf{R}}{dt} \). For the video frames, however, the radius and time are not continuous variables, so we can only find the average velocity between two frames. That is, \( \mathbf{v} = \frac{\Delta \mathbf{R}}{\Delta t} \). To find the average velocity at frame 4 (time=0.4s), we’ll find the change in position between frame 3 (time=0.3s) and frame 5 (time=0.5s). On the graph above, draw (to scale) the vector \( \Delta \mathbf{R} = \mathbf{R}_5 - \mathbf{R}_3 \). Because the time between each frame is 0.1s, \( \Delta t = 0.2s \).

1. Calculate the plane’s average velocity at frame 4. First calculate the components of the velocity vector, and then find its magnitude and the angle the vector makes with the positive \( x \)-axis.

Components:

\[
\begin{align*}
v_{x4} &= \frac{(x_5 - x_3)}{(0.2s)} = \underline{\text{_____}} \text{ m/s} \\
v_{y4} &= \frac{(y_5 - y_3)}{(0.2s)} = \underline{\text{_____}} \text{ m/s}
\end{align*}
\]

Magnitude and direction:

\[
\begin{align*}
|\mathbf{v}_4| &= \underline{\text{_____}} \text{ m/s} \\
\theta_{v4} &= \underline{\text{_____}} ^\circ
\end{align*}
\]

Check to make sure that your answer for the angle is in agreement with your drawing of \( \Delta \mathbf{R} \). Draw (to scale) \( \mathbf{v}_4 \) on your graph on page 5.

Click on **Data Analysis** and have the computer plot velocity versus time. View the graph. Note that the graph has three curves. Two are the components of the velocity and one the magnitude of the velocity vector, or the speed of the plane. The small tail on the “V” shows the direction of the vector.

2. Get the computer’s calculation for the two components of the velocity at frame 4 (time=0.4s) from either the graph or the data table.

\[
\begin{align*}
v_{x4} &= \underline{\text{_____}} \text{ (m/s)} \\
v_{y4} &= \underline{\text{_____}} \text{ (m/s)}
\end{align*}
\]

Compare these results with your values for the components of \( \mathbf{v}_4 \). Does the computer’s calculation of the plane’s speed agree with yours within 5%? If they are not in good agreement, check your calculations.

3. Percent difference between \( v_{x4} \) calculated and computer’s value: \( \underline{\text{_____}} \)

Percent difference between \( v_{y4} \) calculated and computer’s value: \( \underline{\text{_____}} \)
E. The plane’s average speed

1. Calculate the plane’s average speed from the distance it moved while making one circle using your values of the radius and period.

\[ v_{\text{average}} = \frac{2\pi R}{P} = \underline{\phantom{0}} \text{m/s} \]

The speed found this way should be in good agreement with the speed found using the derivative of the position vector (section D). If they do not agree check your calculations.

2. Percent difference between \( v_{\text{average}} \) and \( |v_4| \): \underline{\phantom{0}}

F. The plane’s acceleration

Just as in the case with velocity, the average acceleration is given by \( \mathbf{a} = \Delta \mathbf{v}/\Delta t \).

1. Calculate the components of the acceleration vector of the plane in frame four, and then find its magnitude and the angle the vector makes with the positive x-axis. You can get the needed information on velocity either from the graph or the data table.

Components:

\[ a_{x4} = \frac{(v_{x5} - v_{x3})}{0.2 \text{ s}} = \underline{\phantom{0}} \text{m/s}^2 \]
\[ a_{y4} = \frac{(v_{y5} - v_{y3})}{0.2 \text{ s}} = \underline{\phantom{0}} \text{m/s}^2 \]

Magnitude and direction:

\[ |\mathbf{a}_4| = \underline{\phantom{0}} \text{m/s}^2 \]
\[ \theta_{a4} = \underline{\phantom{0}}^\circ \]

2. Draw (to scale) \( \mathbf{a}_4 \) on your graph on page 5.

Click on Data Analysis and have the computer analyze the plane’s acceleration. View the graph of acceleration as a function of time. Again, the graph has three curves. Two are the components of the acceleration and one the magnitude of the acceleration vector. You should change the vertical scale of the graph so you can study the acceleration more easily.

3. Get the computer’s calculation for the two components of the acceleration at frame 4 (time = 0.4s) from either the graph or the data table.

\[ a_{x4} = \underline{\phantom{0}} \text{ (m/s}^2) \]
\[ a_{y4} = \underline{\phantom{0}} \text{ (m/s}^2) \]

The computer’s value for the acceleration found should be in good agreement with the acceleration you calculated above. If they do not agree check your calculations.

4. Percent difference between \( a_{x4} \) calculated and computer’s value: \underline{\phantom{0}}

Percent difference between \( a_{y4} \) calculated and computer’s value: \underline{\phantom{0}}
G. The plane’s average acceleration

1. Calculate the plane’s average centripetal acceleration from its average speed and radius.

\[ a_{\text{average}} = \frac{v_{\text{average}}^2}{R} = \text{________} \text{m/s}^2. \]

If the plane’s speed is constant, then the acceleration should have no components in the direction of the speed, that is, tangent to the orbit. The acceleration should be purely centripetal. The acceleration should then be in good agreement with the acceleration you found using the derivative of the velocity vector in section F. If they do not agree at least approximately, check all your calculations.

2. Percent difference between \( a_{\text{average}} \) and \( |a_4| \): \_________________________