Team: ____________________

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Force and Motion 2

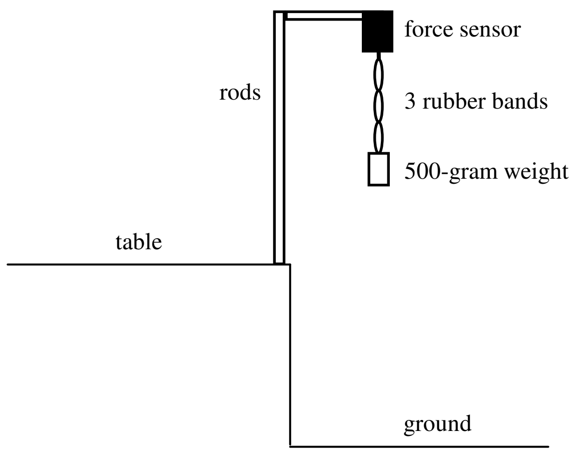
In the first *Force and Motion* lab, you studied constant forces and friction-free motion. In this sequel, you will study forces that depend on time and position. You will also explore the force of friction.

Part I. Force Curves

A *force curve* is the graph of the force **F** on an object as a function of time t. Here, you will use the *force sensor* to measure the *force function* **F**(t) for a "bungee jump" and a collision. "Reading" the shape of force curves is an integral part of the "art" of Newtonian mechanics.

A. BUNGEE JUMP: What Kind of Forces Do You Feel During a Bungee Jump ?

Build the following small-scale replica of a bungee-jump system:



Open the file *ForceProbe*. Set the switch on the sensor to the '10N' range. *Zero* the sensor when it is in the vertical position and nothing is attached to the hook.

Start with the person (weight) high above the ground (floor, not table) so that the bungee cord (rubber bands) is relaxed (unstretched). Caution: Before releasing the heavy 500-gram mass, make sure it will not hit your foot. Activate the force sensor. Release the person. The force sensor will record the tension force F(t) exerted by the bungee cord on the person. Change the scales of F and t on your F(t) graph (try the Autoscale) in order to magnify all features and clearly display the shape of the force curve during the entire sequence of up-and-down motions of the bungee jump.

PRINT your force curve.

1. Mark the two points on your printed force curve where the person is momentarily at rest at the extreme *bottom* (closest to ground after being released) and at the extreme *top* (farthest from ground after the first 'bounce') of the bungee jump. Label the points *"Bottom of Jump"* and *"Top of Jump"*.

2. What is the <u>*Apparent Weight*</u> of the person at these two extreme points?

Note: The force sensor is a "hanging weight scale" just as a bathroom scale is a "standing weight scale". The weight of any object hanging from the force sensor is simply the value (F) recorded by the sensor. If the object is not accelerating, then the sensor reads the "actual weight" (F=W). If the object is accelerating up or down, then the sensor reads the "apparent weight" ($F>W$ or $F<W$).

Write the values of the two apparent weights next to the corresponding two extreme points on your force curve.

3. Mark the value of the actual weight of the person on the *Force* (Newton) axis. Draw a horizontal line at this value across the entire graph. Write *"Actual Weight"* on the line.

4. At the bottom of the bungee jump, the jumper feels ______ times heavier/lighter (circle one) than their actual weight.

B. COLLISION: What Does the Force of Impact Look Like During a Collision ?

Collisions are everywhere. A bat hits a ball. A foot kicks a ball. A car hits a car. A ball bounces off the floor. A rusher tackles a quarterback. The wind hits your face. An electron scatters off a proton. An asteroid hits the earth.

Place the force sensor against the end of the track. Start the cart at the other end. Zero the force sensor. Activate the force sensor. Give the cart an initial velocity. Let it coast along the track and collide with the force sensor (hook).

Change the F and t scales on your F(t) graph so that your "collision triangle" fills the entire graph window. PRINT your force curve.

1. Find the <u>maximum force</u> $\mathbf{F_{max}}$ during the collision, i.e. What is the force that occurs between the cart and sensor when these two objects are in closest contact and their *"atomic bonds"* are maximally compressed ?

2. Find the <u>collision time</u> Δt, i.e. What is the interval of time during which the cart and sensor are in "contact" ?

Mark the value of "F_{max}" on the F axis. Mark the time interval "Δt" on the t axis. Report the values of these collision parameters here:

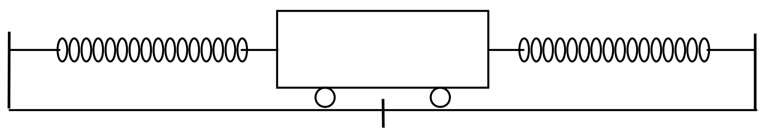
F_{max} = __________ *Newtons .* Δt = __________ *milliseconds .*

Part II. Position-Dependent Force – A Case Study

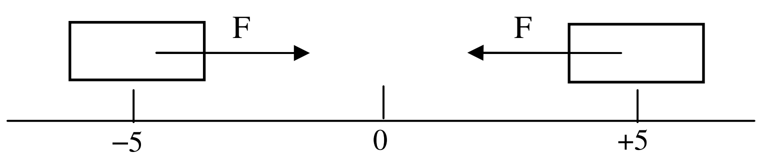
Feel the Force. How does F(x) depend on x ?

Many forces in nature depend on *position*. The gravitational force on a planet depends on its distance from the sun. The magnetic force on a mag-lev train depends on its distance above the tracks. The electric force on an electron in an atom depends on its distance from the protons in the nucleus.

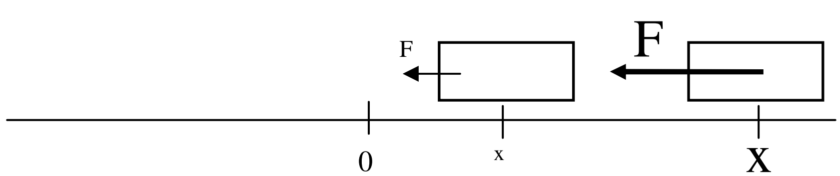
In this experiment, you will encounter one of the most important position-dependent forces in physics: the force due to a spring. The mechanical system that you will study consists of a cart connected to two springs moving on a metal track:



The origin of the x-axis is chosen to be the point where the net force on the cart is zero. The cart will remain at rest at x = 0. At this so-called "equilibrium point", the left spring pulls on the cart with a force that exactly balances the pull of the right spring. Thus F = 0 at x = 0. With your hand, pull the cart away from x = 0 to another position, such as x = +5 cm and x = −5 cm, and *feel the force*. The force always restores the mass back toward x = 0:



As you pull the cart *further* away from x = 0, the restoring force back to x = 0 becomes *stronger* due to the increasing expansion and compression of the springs. Thus the magnitude of F is <u>not</u> constant: *F depends on x.*



Since the force F on the cart depends on the position x of the cart, the physical quantity F is represented by a *mathematical function* F(x). Given any value of x, the function F(x) specifies the value of the force at that particular x. Based on your qualitative study so far, all you know about this function is that F(0) = 0 and that the magnitude of F(x) is an increasing function. Your experimental goal is to find the precise form of this force function. Does F(x) have the form $F = 3x$ or $F = 5x^2$ or $F = 10^x$ or $F = \ln(1+x)$? You will find F(x) two different ways:
(1) Dynamically, by observing the back-and-forth motion of the cart. (2) Statically, by using a spring scale.

1. Discovering the Law of Force

In physics, a *force law* is a mathematical rule that specifies how force depends on position. The method used by physicists to discover the force laws of nature is simple: *Observe the Motion , Deduce the Force*. Here are two examples: (1) Observe the motion of the moon in the cosmos, Deduce the law of gravity. (2) Observe the collision of quarks in a particle accelerator, Deduce the law of nuclear force. In this experiment, you will

> Deduce how F(x) depends on x – the *Law of Elastic Force* − by measuring how the position x(t) of the cart depends on time t

A. Sinusoidal Motion

Open Logger Pro file *Moving Along*. Level the track using the "steel ball test". Place the wooden block covered with felt on top of the cart. This block helps the sensor "see" the cart. Place the motion sensor on the table off one end of the track. The sensor should stand in a tall vertical orientation. Pull the cart from x = 0 to about x = 20 *cm*. Release the cart and record its motion.

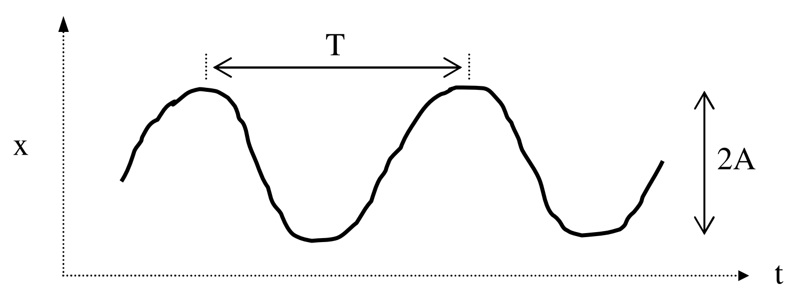
Does the x(t) curve look like a *"sine curve"* ? A sine it is!

There is a deep connection between *force* and *geometry*. In the past you studied the connections "zero force ⇔ linear motion" and "force of gravity ⇔ parabolic motion". Now you will study the connection "force of elasticity ⇔ sinusoidal motion".

The mathematical equation describing the x-t curve recorded by your motion sensor is

$$x(t) = A \sin(2\pi t/T) .$$

This sinusoidal *worldline* is graphed below.



The motion parameters, A and T, specify the overall shape of the worldline: the "height" and the "width" of the sine curve. More precisely, the *amplitude* A is the maximum displacement of the cart away from the equilibrium point. The *period* T is the time it takes the cart to complete one oscillation (one back-and-forth motion) Note that the function $x(t) = A\sin(2\pi t/T)$ is bounded, $-A \le x \le A$, and periodic, $x(t+T) = x(t)$.

Examine the x-t curve recorded by your motion sensor. Read off the values of A and T that characterize the sinusoidal motion of your cart:

A = _____________ m . T = _____________ s .

B. The Force That Causes Sinusoidal Motion

Calculus Exercise Given: Position $x(t) = \sin 2t$. Mass $m = 3$.
Prove: Force $F(x) = -12x$.

Deriving the Force Function. The Force Constant.

In the calculus exercise above, you proved that a linear force function $F \sim x$ causes sinusoidal motion $x \sim \sin t$. It is easy to generalize this result using symbols (m, A, T) instead of numbers for the system parameters. Here is the result:

If $x = A \sin(2\pi t/T)$ is substituted into Newton's law $F = m\, d^2x/dt^2$, then the resulting force function is

$$F(x) = -\left(\frac{4\pi^2 m}{T^2}\right) x .$$

Physicists write this force law in the form

$$F = -kx ,$$

where the force constant k is

$$k = 4\pi^2 m/T^2 .$$

The magnitude of the force F on the cart increases in direct proportion to the distance x that the cart is displaced from the equilibrium point (x=0). If you double x , then you double F. The constant k is the proportionality constant between F and x. The minus sign in $F = -kx$ indicates that the force is always opposite to the displacement.

Note that the force-period relation is $F \sim 1/T^2$. This relation makes intuitive sense. If the cart moves back and forth rapidly (small T), then the velocity changes rapidly (large a) and thus the force must be strong , i.e. small T $\Rightarrow$ large $a \sim x/T^2$ $\Rightarrow$ large $F = ma$.

Physicists say that the force of elasticity is a **"Linear Force"** because the approximate force function $F = -kx$ is a *linear* function of x. In contrast, the forces of gravity and electricity are exact **"Inverse-Squared Forces"** ($F = k/x^2$). Whereas the spring force gets stronger as the distance increases, the forces of gravity and electricity get weaker as x increases.

C. Measuring the Force Constant with a Stopwatch

1. If you know the value of the force constant k, then you know everything about the system dynamics.
2. To find the force constant in $F = -(4\pi^2 m/T^2)\,x$, you need to measure the mass m of the system and the period T of the motion.

Measure m of system (cart + block). m = ______________ kg .

Measure the period with a stopwatch. To reduce the relative error, measure the time it takes the cart to complete five oscillations (five back-and-forth motions). Divide this five-cycle time by five to obtain the period. Report the value of the period of your oscillating cart.

T = _______________ s .

Is this stopwatch value of T consistent with (within 5% of) the graphical value of T that you obtained from the "width" of your x-t curve? If not, see your instructor.

Based on your measured values of m and T, what is the force constant that characterizes your spring system? Show your calculation.

k = _______________ N/m.

Given this value of k, compute the *magnitude* of the force ($F = -kx$ without the minus sign) at those values of x listed in the following table.

Force deduced from the Motion

x (*m*)	0.00	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
F = kx (*N*)	0								

2. Measuring F(x) with a Spring Scale

Now that you have deduced the force function F(x) from the motion data x(t), you can check your result by directly measuring F(x) using a spring scale. This may seem like an easy method to find the force, but remember, someone had to calibrate the force meter to convert a length measurement (stretch of spring) into a force quantity (Newtons). Finding force from F = *ma* is the *gold standard* for measuring force.

First make sure the spring scale is "zeroed" in the horizontal position. Start with the cart at rest at x = 0. The two springs should still be connected to the ends of the cart and the track. Attach the spring scale to the right end of the cart. While keeping the scale as horizontal as possible (parallel to the track), pull on the scale case and displace the cart from x = 0 to x = 0.02 m. Hold the cart at 0.02 m and record the scale reading F_s in the table below. Next pull the cart to 0.04 m, hold, and record the reading, etc. Note that since the cart is at rest at each location, the force exerted on the cart by the scale is equal and opposite to the force exerted on the cart by the two attached springs. The subscript "s" on F_s denotes that this force is measured with the spring scale.

Force measured with the Spring Scale

x (*m*)	0.00	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
F_s (*N*)	0								

Use the program *Graphical Analysis* to plot F_s versus x (F_s on the y axis). Your data points should fall on a line. Analyze the graph to find the best-fit line through the points. The equation of the line is $F_s = k_s x$. Report the value of the slope:

k_s = _________________ N/m .

On the same graph, plot the force function F = kx that you discovered from your analysis of the oscillatory motion (see the Table *Force deduced from the Motion*). [How do you graph a second set of data on the same graph? Click on "Data" to enter Data Set 2. Click on the "y" label of the graph to plot Data Set 2] Note that since the minus sign in the force law F = −kx has been omitted in both graphs, these graphs display the magnitude of the force and not the direction.

PRINT your graph On the printed graph, label the F = kx line with the name "*Force deduced from the Motion*". Label the $F_s = k_s x$ line with the name "*Force measured with the Spring Scale*".

3. Compare Your Measured Force Laws

The force function F(x) = − kx deduced by *analyzing the oscillatory motion* is

F(x) = − _________ x .

The force function $F_s(x) = -k_s x$ obtained by directly *reading the spring scale* is

F(x) = − _________ x .

The percent difference between the force constants k and k_s is ______ % .

Part III. Exploring Friction

The force of kinetic friction acts on any object sliding across a surface. This friction force f obeys the empirical law:

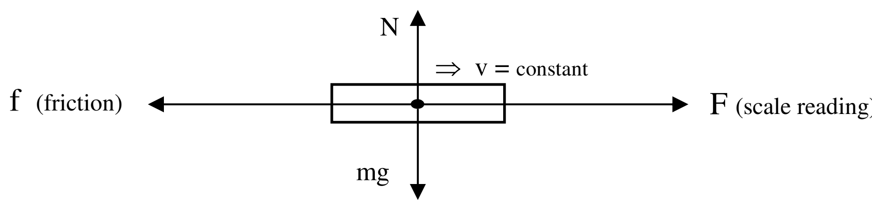
$$f = \mu N ,$$

where N is the normal force and μ is the coefficient of kinetic friction. The coefficient μ depends on the nature of the two surfaces in contact. For steel on steel, $\mu = 0.5$. For Teflon on steel, $\mu = 0.04$. For rope on wood, $\mu = 0.3$. For shoes on ice, $\mu = 0.05$. For rubber on dry concrete, $\mu = 0.8$. For rubber on wet concrete, $\mu = 0.3$.

Measuring the Coefficient of Friction

You goal is to measure the value of μ for felt on aluminum. Attach the spring scale to the wooden block covered in felt. Practice pulling the block across the aluminum track at a constant speed. To a good approximation, the force of friction is independent of speed.

Why pull at constant speed? If the speed of the block is constant, then its acceleration is zero, which means that the *net force* on the block is zero, which means that the scale reading F (spring force) equals the magnitude of the friction force f .



Place 200 *grams* on top of the block. Pull the block at three different constant speeds: "slow", "medium", and "fast". For each speed, observe the scale reading and compute the coefficient of friction in the space below:

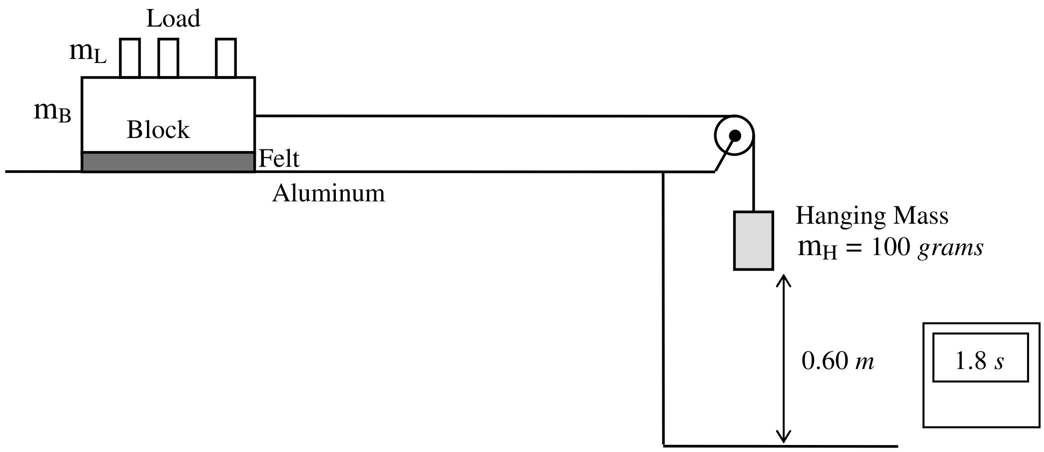
Slow speed μ = ________. Med speed μ = ________ . Fast speed μ = _______.

See your instructor if your three values of μ differ by more than 10%. Average your values.

μ = ______________ (*felt* on *aluminum*) .

Part IV. Experimental Design: Lowering a Weight 3/5 *m* in 9/5 *s*.

Your goal is to lower a "heavy" weight (100 *gram* brass cylinder) over a pulley so that the weight, starting from rest, falls to the ground – a distance of 0.60 *meters* – in a time of 1.8 *seconds*. Here is the schematic of the mechanical system:



The Theory

1. Calculate the value of the acceleration a of the hanging mass as required by the *design specs* (falls 0.60 *m* in 1.8 *s*).

a = ___________ m/s^2.

2. Draw a free body diagram for the hanging mass m_H. Set up Newton's equation of motion $\mathbf{F}_{net} = m_H \boldsymbol{a}$ for this body. Plug the numerical values of m_H and a into Newton's equation and solve for the value of the tension T in the string.

T = _____________ *N* .

3. Treat the Block-plus-Load system as *one body* of mass $m \equiv m_B + m_L$. Draw a free body diagram for the mass m. Set up Newton's equation $\mathbf{F}_{net} = m\boldsymbol{a}$ for m. Plug the numerical values of T , μ , a , and g into Newton's equation and solve for the value of m. Then compute the mass of the load using the relation $m = m_B + m_L$.

m = ___________ *kg*. m_B = ___________ *kg*. m_L = ____________ *kg*.

* Have your instructor check your *theoretical work* before you move on to the *experimental stage* of the design project.

The Experiment

Check that the track is level – again! This design experiment is sensitive to track tilt. Also make sure that the string is horizontal (if it is not horizontal, then adjust the tilt of the pulley).

Add the amount of load m_L to the block as predicted by your theory. Get the mass accurate to within one or two grams. Distribute the load mass evenly over the top of the block. Start the hanging mass at the specified height (0.60 *m*) above the ground. Release the system from rest and measure the time it takes (using a *stopwatch*) for the hanging mass to hit the ground. Repeat the measurement five times and compute the average value t.

Time trials: ________ ________ ________ ________ _________

t = ____________ *seconds* .

How does your value of t compare with the design specifications of 1.8 *seconds* ?