Energy is a vital concept in science. Energy can exist in several forms, including mechanical energy, electrical energy, thermal energy, chemical energy, and nuclear energy. Unlike the everyday concepts of mass, velocity, and force, the concept of energy is less intuitive. What exactly is energy? Is it a “thing”? Where is it located? Does it move around? You have used various scales, sensors, and probes to measure mass, velocity, and force. Does there exist an energy sensor?

Energy is the single concept that unifies all science. It plays a key role in understanding physical, chemical, and biological processes, such as star formation, chemical reactions, photosynthesis, and the production of heat, light, sound, and electricity. It is difficult for a physicist, a chemist, or a biologist to get through their working day without saying the word “energy”. The Conservation of Energy is one of the “top five” most important universal principles of nature.

Part I. Work and Energy

Work is “force acting over distance”. If a constant force \( F \) acts on an object that moves a distance \( d \), then the work \( W \) done by the force is defined by the relation

\[
W \equiv F \cdot d .
\]

Recall that the dot product is \( F \cdot d = Fd \cos \theta \), where \( \theta \) is the angle between the vectors \( F \) and \( d \). If the force varies with position \( x \), then the total work \( W = \int Fdx \) is a sum (integral \( \int \)) over the incremental works \( dW = Fdx \). The concept of work is the key that opens the door to the world of energy. All the properties of energy can be traced back to the notion of work.

Kinetic Energy is the “energy of motion”. If an object of mass \( m \) is moving with a velocity \( v \), then the kinetic energy \( K \) of the object is

\[
K \equiv \frac{1}{2} mv^2 .
\]

The quantity \( \frac{1}{2} mv^2 \) is important in physics because, as you will discover, it is the unique property of motion that measures the effect of doing work. Whereas \( mv \) and \( mv^2 \) occupy a special place in physics, the quantities \( mv^3 \) and \( mv^4 \) are irrelevant.

The Work-Energy Theorem is the deep relation between work and kinetic energy:

\[
W = \Delta K .
\]

The net work done on an object is equal to the change in the kinetic energy of the object. This theorem is a consequence of Newton’s law of motion \( F = ma \). In essence, \( W = \Delta K \) results from compounding both sides of \( F = ma \) over a displacement: \( \int [F = ma] \, dx \). The left side \( \int Fdx \) is \( W \) and the right side \( \int mv dv \) is \( \Delta \frac{1}{2} mv^2 \). In short, \( W = \Delta K \) is the energetic version of \( F = ma \).
Note that $F = mv/\Delta t$ says that $F\Delta t = \Delta mv$, while $W = \Delta K$ says that $F\Delta x = \Delta \frac{1}{2}mv^2$. Here is Newtonian mechanics in a nutshell:

A force $F$ acting over a *temporal* interval $\Delta t$ changes the *momentum* $mv$.

A force $F$ acting over a *spatial* interval $\Delta x$ changes the *kinetic energy* $\frac{1}{2}mv^2$.

### A. Experimental Proof of the Work-Energy Theorem (for a Constant Force)

In this experiment, you will exert a constant force over a certain distance, thereby performing a constant amount of work and causing a change in kinetic energy. First make sure that the spring scale is “zeroed”. Attach the spring scale to the cart and pull the cart, starting from rest, over a distance of 0.80 m along the level track with a constant force of $F = 0.10$ N.

Maintaining a constant force as you pull may be a bit tricky. Remember, the force is constant, not the speed. The speed should increase at a steady rate. It is okay if the force fluctuates somewhat around the value 0.1 N. Sometimes you pull to hard and other times too soft, so the plus and minus errors tend to cancel each other out! While you are pulling the cart over the 80 cm distance, you partner needs to measure the time it takes to cover this distance with a stopwatch. Your team should practice this pulling and timing. Repeat the experiment five times to get a sense of the “precise value” of the time. List your measured times and report the average value.

$t = \underline{\phantom{10000}}\text{ s}$.

The initial velocity $v_o$ of the cart is zero. Find the final velocity $v$ from your measured value of $t$ using the kinematic relation, $d = \frac{1}{2}(v_o+v)t$. Show your calculation of $v$.

$v = \underline{\phantom{10000}}\text{ m/s}$.

Measure the mass of the cart.

$m = \underline{\phantom{10000}}\text{ kg}$.

Calculate the final kinetic energy of the cart.

$\frac{1}{2}mv^2 = \underline{\phantom{10000}}\text{ J}$.
Calculate the work done by the force pulling the cart.

\[ Fd = \underline{\text{\ }} \text{J}. \]

Based on your experimental results, can you conclude that the Work-Energy Theorem is valid? To within what percent error did you prove the theorem?

**B. Experimental Proof of W = ΔK (for a Varying Force)**

The varying force will be due to a spring. First measure the work done to stretch the spring. Attach one end of the spring to the horizontal rod mounted above the table so that the spring hangs vertically. Hang a small brass weight (about 20 g) on the other end of the spring and measure the distance \( x \) the spring stretches. Add more mass and measure the stretch. Altogether, use five different masses that range between 20 g and 150 g. Note that the stretch distances \( x \) are measured from the same reference point (\( x = 0 \)) each time. Record the forces (\( F = mg \)) and the stretches (\( x \)) in the table.

<table>
<thead>
<tr>
<th>( x ) (m)</th>
<th>0</th>
<th>( m )</th>
<th>( x' )</th>
<th>( m' )</th>
</tr>
</thead>
</table>

Use the program *Graphical Analysis* to graph \( F \) versus \( x \). Print the graph. Find the work required to stretch the spring from \( x = 0.15 \) m to \( x = 0.30 \) m. The easiest way to find the work is to compute an area. Recall that the geometric meaning of \( W = \int F \, dx \) is \( W = \text{area under } F-x \text{ curve} \). Compute this area by hand *directly on your printed graph*. Note that the area region consists of a rectangle and a triangle.

\[ W = \underline{\text{\ }} \text{J}. \]
Attach one end of the spring to the cart and the other end to the track:

Note that the origin (x = 0) is the position of the cart when the spring is relaxed (unstretched) and horizontal. Pull the cart to x = 0.30 m and release. The spring force will now do work on the cart. The speed of the cart will increase. Measure the speed v of the cart when it is at x = 0.15 m using the motion sensor. [Open Logger Pro file Changing Velocity 1. In order to magnify the good data region, change the time scale in the graph window so that it spans the range from 0.0 to 3.0 seconds] Remember that the motion sensor measures distances from the front face of the sensor. Thus in order to find v from the motion-sensor data, you will need to measure the distance (using a meter stick) between the sensor and the cart when the cart is at x = 0.15 m. Run the experiment again to make sure that your measured value of the velocity is reliable. If your two measured velocities differ by more than 5%, repeat the experiment.

You have already measured the mass m of the cart. Compute the kinetic energy K of the cart at x = 0.15 m. Record your values of m, v, and K:

\[
m = \text{______________ kg}.
\]
\[
v = \text{______________ m/s}.
\]
\[
K = \text{______________ J}.
\]

Are your results consistent with the Work-Energy Theorem? Show the numbers you are comparing. What is the percent difference?

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Part II. Conservation of Energy

The Essence of Energy
There are two forms of mechanical energy: *kinetic* and *potential*. Kinetic energy $K(v)$ is the *energy of motion* – the ability of an object to do work by virtue of its *velocity* $v$. Examples include a hammer with speed $v$ about to hit a nail, a bowling ball with speed $v$ about to hit bowling pins, and a meteor with speed $v$ about to hit the earth. Potential energy $U(x)$ is the *energy of position* – the ability of an object to do work by virtue of its *position* $x$. Examples include a spring compressed a distance $x$ and a weight elevated a height $x$.

We have already encountered kinetic energy: $K(v) \equiv \frac{1}{2} mv^2$. The potential energy $U(x)$ associated with a conservative force $F$ is defined by the relation

$$U(x) - U(0) \equiv -W(0 \rightarrow x),$$

where $W(0 \rightarrow x)$ is the work done by $F$ when the object moves from 0 to $x$. In short, $U$ is defined by the relation $\Delta U = -W$. In terms of the potential energy, the work-energy theorem ($W = \Delta K$) is $-\Delta U = \Delta K$ or $\Delta K + \Delta U = 0$ or $\Delta (K+U) = 0$. The mechanical energy $E$ is defined to be the sum of the kinetic and potential energies:

$$E \equiv K + U.$$

In terms of $E$, the work-energy theorem $\Delta (K+U) = 0$ says that $\Delta E = 0$ and thus

$$E_{\text{initial}} = E_{\text{final}}.$$

This is energy conservation. During any mechanical process (state $i \rightarrow$ state $f$) due to conservative forces, the energy does not change ($E_i = E_f$). This is a remarkable feature of the motion. During the motion, everything (x, v, K, U) is changing in time – except for $E$! The energy can transform between the two forms, K and U, but the total K+U must remain constant. Energy cannot be created or destroyed.

Introducing a quantity that does not change in time is the essence of all conservation laws in physics. Searching for conserved quantities in the universe is big business among professional physicists. In addition to the conservation of energy, there is the conservation of momentum and the conservation of mass. In the theory of relativity, mass and energy are equivalent. The motion of a mechanical system can be very chaotic with so many system variables changing in time. Conservation laws bring an order to the chaos.

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**The System**

You will study energy conservation in a system consisting of a cart of mass $M$ connected via a string to a 20 gram mass $m$ hanging over a pulley. Here is a schematic of the system:
Note that $x$ is the position of the cart relative to the front face of the motion sensor, and $y$ is the position of the hanging mass relative to the ground.

**The Motion**

Make sure the track is level. There is a leveler on the front table. Start the system in the following configuration: The initial position of the hanging mass is exactly at $y = 50$ cm and the initial position of the cart $x$ is somewhere between 20 cm and 30 cm. This initial condition determines the length of the string that you must use to connect the cart and the hanging mass. Note that $x$ and $y$ are related.

Starting with this initial configuration, release the cart from rest and measure its motion with the motion sensor. [Open Logger Pro file *Changing Velocity 1*. The time scale in the graph window should go from 0.0 to 3.0 seconds] Find the velocity $v$ of the cart at five different times during the motion—when the hanging mass is at $y = 50$ cm, 40 cm, 30 cm, 20 cm, 1 cm.

Remember, since the motion sensor gives the velocity as a function of $x$, you need to find the five $x$ values corresponding to the five given $y$ values. To find the value of $v$ at one of your $x$ values by reading the motion-sensor data table, you may have to extrapolate between two neighboring entries in the table. Record your distances and velocities in the following motion table:

<table>
<thead>
<tr>
<th>$y$ (m)</th>
<th>$x$ (m)</th>
<th>$v$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>0.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**The Energy**

Calculate the kinetic energy $K$, the potential energy $U$, and the mechanical energy $E$ of the system (cart plus hanging mass) at these five different times during the motion. Let the potential energy $U(y)$ of the system be zero when the hanging mass is on the ground ($y = 0$). This defines the zero reference point: $U(y=0) = 0$. As a “certified energy accountant” (CEA), fill in the following energy table:

<table>
<thead>
<tr>
<th>$y$ (m)</th>
<th>$K$ (J)</th>
<th>$U$ (J)</th>
<th>$E$ (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Show a sample calculation of \( K \), \( U \), and \( E \) for \( y = 0.30 \) m.

Does the “energy book” balance?

What is happening to \( K \), \( U \), and \( K+U \) as time goes on? Make a qualitative plot of these three energies on the following energy-time graph. Label the three curves \( K \), \( U \), and \( E \).

What are the changes in the kinetic and potential energies, \( \Delta K \) and \( \Delta U \), between \( y = 0.40 \) and \( y = 0.20 \)? Interpret your answer.

Estimate the wattage of this “machine” – the average rate (Joules per second) at which potential energy is converted into kinetic energy as the hanging mass falls. Hint: Estimate how long it takes for the mass to hit the ground.
Was any energy created or destroyed during the motion? Explain.

Based on your measured values of $E$, what would you say is the value of the conserved energy of your system – the immutable, unchanging, invariant, enduring number of Joules contained in the system during the motion. Like all experimental values, report your value as number plus or minus an uncertainty.

$$E = \underline{\phantom{0000}} \pm \underline{\phantom{0000}} \text{ J}.$$ 

**Part III. Designing a Bungee Jump System**

A schematic of the system appears below. A 200 gram mass (the “person”) hangs from two silver springs in series (the “bungee chord”).

![Bungee Jump Schematic](image)
Your design goal is to find the starting height $H$ above the ground (floor) so that the mass, when released from rest, falls downward and stops at a point that is just 10.0 cm above the ground, before being pulled back up by the stretched springs. Note that in the starting position, the spring is in its relaxed (unstretched) state.

**Theory**

System parameters:

- $m =$ mass of “person”.
- $k =$ spring constant of “bungee chord.”
- $H =$ starting height above the ground.
- $d =$ distance above the ground where the person momentarily stops.
- $g =$ acceleration due to gravity near the surface of the earth.

In the space below, derive the theoretical formula that gives $H$ in terms of $m$, $k$, $d$, $g$. Work with the symbols, not the numbers.

$$H = \text{______________________________}.$$

**Experiment**

Measure the value of $k$ that characterizes the elasticity or “springiness” of the bungee chord. It is perfectly valid to treat the two-spring system as one spring whose force constant is $k$. Explain your procedure. Provide all data and graphs.

$$k = \text{_____________________ N/m}.$$
Calculate the predicted value of $H$ by inserting your measured value of $k$ and the known values of $m$, $d$, and $g$ into your theoretical formula for $H$. Show your calculation.

Predicted $H = \text{____________________ m}$.

*Performing the Bungee Jump*

Hook the 200 gram mass to the end of the spring. Before you let the mass fall, make sure that the spring is in its relaxed (unstretched) state. This is the initial condition. As a team, one of you should be at the starting level to release the mass, while the other should be at the ground level to see where the mass stops – hopefully before it hits the ground!

*Results of the Jump*

If your bungee jump was successful, how far from the ground was the mass when it momentarily came to rest?

actual $d = \text{______________ cm}$.

What is the percent difference between your actual $d$ and the design goal $d = 10.0$ cm? What are the sources of errors that could account for the discrepancy?