Energy

Energy is the central concept in science. Energy can exist in several forms, including mechanical energy, electrical energy, thermal energy, chemical energy, and nuclear energy. Unlike the everyday concepts of mass, velocity, and force, the concept of energy is less intuitive. What exactly *is* energy? Is it a "thing"? Where is it located? Does it move around? You have used various scales, sensors, and probes to measure mass, velocity, and force. Does there exist an *"energy sensor"* ?

Energy is the single concept that unifies all science. It plays a key role in understanding physical, chemical, and biological processes, ranging from quantum jumps in atoms, to photosynthesis in plants, to the formation of stars in the universe. Energy explains how heat, light, sound, and electricity can transform into one another. It is difficult for a physicist, a chemist, or a biologist to get through their working day without saying the word "energy". The *Conservation of Energy* is one of the "top five" most important universal principles of nature.

Part I. Work and Kinetic Energy

Work is *"force acting over distance"*. If a constant force **F** acts on an object that moves a distance **d** , then the work W done by the force is defined by the relation

$$W \equiv \mathbf{F} \bullet \mathbf{d} \ .$$

Recall that the dot product is $\mathbf{F} \bullet \mathbf{d} \equiv Fd\cos\theta$, where θ is the angle between the vectors **F** and **d**. If the force varies with position x, then the total work $W = \int F dx$ is a sum (integral $\int$) over the incremental works $dW = Fdx$. The concept of work is the key that opens the door to the world of energy. The meaning of energy and all its properties can be traced back to the notion of work!

Kinetic Energy is the *"energy of motion"*. If an object of mass m is moving with a velocity v, then the kinetic energy K of the object is

$$K \equiv \tfrac{1}{2} mv^2 \ .$$

The quantity $\frac{1}{2} mv^2$ is important in physics because, as you will discover, it is the unique property of the motion that measures the *effect* of doing work. Whereas mv and mv^2 occupy a special place in physics, the quantities mv^3 and mv^4 are irrelevant.

The Work-Energy Theorem is the deep relation between work and kinetic energy:

$$W = \Delta K \ .$$

The net work done on an object is equal to the change in the kinetic energy of the object. This theorem is a logical consequence of Newton's law of motion $F = ma$. In essence, $W = \Delta K$ results from compounding (summing) both sides of $F = ma$ over a displacement: $\int [F = ma]\, dx$. The left side $\int F dx$ is W and the right side $\int mv dv$ is $\Delta \frac{1}{2}mv^2$. In short, $W = \Delta K$ is the "*energetic version*" of $F = ma$.

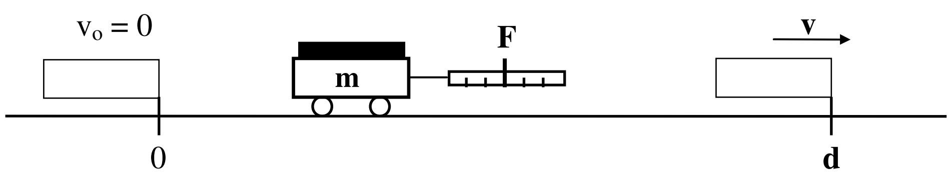
Note that $F = m\Delta v/\Delta t$ says $F\Delta t = \Delta mv$, while $W = \Delta K$ says $F\Delta x = \Delta \frac{1}{2}mv^2$. These two relations represent all of Newtonian Mechanics in a "nutshell":

$F\Delta t = \Delta mv$: *A force* F *acting over a* *temporal* *interval* Δt *changes the* *momentum* mv.
$F\Delta x = \Delta \frac{1}{2}mv^2$: *A force* F *acting over a* *spatial* *interval* Δx *changes the* *kinetic energy* $\frac{1}{2}mv^2$.

A. Experimental Proof of the Work-Energy Theorem (for a Constant Force)

Note: Do not spend more than 5 minutes on this Part A Experiment. Your goal is to get a *kinesthetic* feel for "work = force × distance". A 30% error is okay.

In this experiment, *you* will exert a constant force over a certain distance, thereby performing a certain amount of work and causing a change in kinetic energy. First make sure that the track is level using the "steel ball test". Make sure that that the spring scale is "zeroed" when held in the *horizontal position* and nothing is attached to the small hook. Place the long black bar weight on top of the cart. Attach the small hook of the spring scale to the cart and pull the cart along the track, starting from rest, over a distance of $d = 0.80$ *m* with a constant force of $F = 0.15$ *N*.



Maintaining a constant force as you pull may be a bit tricky – this is why a 30% error is okay. Remember, you want to keep the force constant, not the speed. The speed should increase at a steady rate. The force will fluctuate around the value 0.15 *N*. While you are pulling the cart over the 80 *cm* distance, your partner will measure the time it takes, *starting from rest*, to cover this distance with a *stopwatch*. Your team should practice this pulling and timing. Repeat the experiment five times to get a sense of the "precise value" of the time. List your measured times and report the average value.

t = ________________ s .

The initial velocity v_o of the cart is zero. Find the final velocity v from your measured value of t using the kinematic relation, $d = \frac{1}{2}(v_o+v)t$. Show your calculation of v.

v = ______________ m/s .

Measure the mass of the moving object (cart + bar weight): m = ______________ kg .

From your measured values of F, d, m, and v, calculate the following quantities:

Fd = (*N*) (*m*) = ________________ *N•m*.

½ mv^2 = ½ (*kg*) ($m/s)^2$ = ________________ $kg•m^2/s^2$.

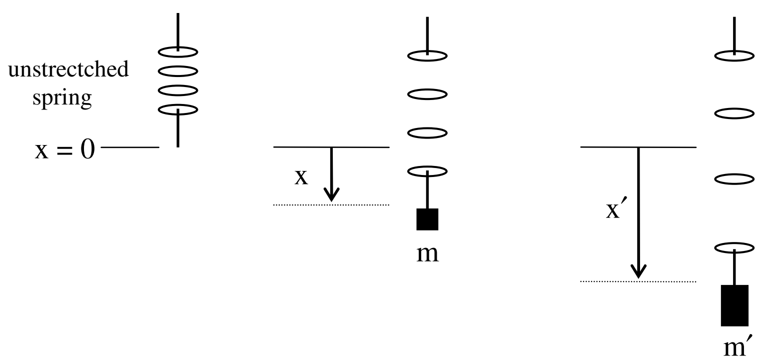
The scientific name of the unit $N•m = kg•m^2/s^2$ is ________________ .

Based on your experimental results, to within what percent error did you prove the *Work-Energy Theorem* ?

B. Experimental Proof of W = ΔK (for a Varying Force)

Measuring Work

The varying force will be due to a spring. Use the wide spring (not the narrow silver spring). First measure the work done to stretch the spring. Attach one end of the spring to the horizontal rod mounted above the table so that the spring hangs vertically. Hang a small (m = 20 *gram*) on the other end of the spring and measure the distance x the spring stretches. Add more mass and measure the resulting stretch.



Note that the stretch distances x are measured from the same reference point (x = 0) each time. Record your measured forces (F = mg) and stretches (x) in the table:

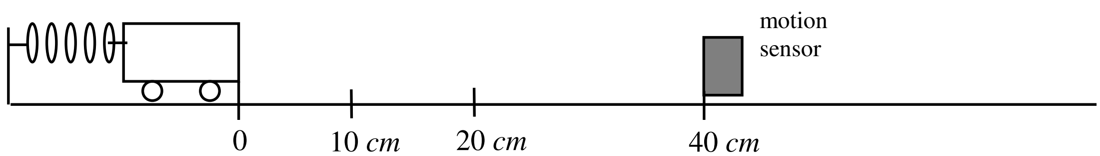
m (*kg*)	0	0.020	0.050	0.070	0.100	0.150
F (*N*)	0					
x (*m*)	0					

Use the program *Graphical Analysis* to graph F versus x (F on the y axis). If your spring obeys Hooke's law (F ∝ x), then your data points should fall on a line. Fit your points with a line. Find the work required to stretch the spring from x = 0.10 *m* to x = 0.20 *m*. The easiest way to find the work is to compute an area. Since the product Fdx is the area of a tiny rectangle of height F and base dx, it follows that the geometric meaning of $W = \int F dx$ is W = *Area Under* F-x *Curve*. Compute the area under your F-x curve by first highlighting the region of your graph between 0.10*m* and 0.20*m* and then clicking the AREA BUTTON. PRINT your graph showing the computer's result (area box). Report your value of W = work done *by* the spring from x = 0.20 *m* to x = 0.10*m*:

W = *area under F-x curve* = _____________ J .

Measuring Kinetic Energy

Attach one end of the spring to the cart and the other end to the track:



The cart should be at rest and the spring should be relaxed – un-stretched, un-bunched, straight and horizontal. This initial condition of the spring is important. Place the motion sensor on the track at a distance of 40 *cm* from the right end of the cart (see picture).

Open Logger Pro file *Changing Velocity* 1. With the cart at rest and the spring relaxed, zero the motion sensor: click *Experiment* and select *Zero*. This sets the origin (x = 0) to be the position of the cart when the spring is relaxed.

Now click *Experiment*, select *Data Collection* and change the Time Length to 3 *seconds* and the Sampling Rate to 100 *samples per second*.

Pull the cart to x = 20 *cm* and release. The spring force will now do work on the cart. The speed of the cart will increase. Measure the speed v of the cart when it is at x = 10 *cm*. To find v when the cart is at 10 *cm*, look at the data table and find a distance value that is closest to x = – 0.10 *m* (the minus sign simply means the cart is displaced *toward* the sensor). Run the experiment again to make sure that your measured values of the velocity are reliable.

PRINT your v-t graph and data table. Underline the value of v in your data table.

You have already measured the mass m of the cart. Compute the kinetic energy K of the cart at x = 0.10 *m*. Record your values of m, v, and K:

m = ______________ kg .

v = _______________ m/s . K = ______________ J .

Conclusion. Are your results consistent with the *Work-Energy Theorem*? Display the numbers that you are comparing and find their percent difference:

Part II. Conservation of Energy

The Essence of Energy. Order out of Chaos.

There are two forms of mechanical energy: *kinetic* and *potential*. Kinetic energy K(v) is the *energy of motion* – the ability of an object to do work by virtue of its *velocity* v. Examples include a hammer with speed v about to hit a nail, a bowling ball with speed v about to hit bowling pins, and a meteor with speed v about to hit the earth. Potential energy U(x) is the *energy of position* – the ability of an object to do work by virtue of its *position* x. Examples include a spring compressed a distance x and a weight elevated a height x.

You have already encountered **kinetic energy**: $K(v) \equiv \frac{1}{2} mv^2$. The **potential energy** U(x) associated with a conservative force F is defined by the relation

$$U(x) - U(0) \equiv -W(0 \rightarrow x) ,$$

where $W(0 \rightarrow x)$ is the work done by F when the object moves from 0 to x. In short, U is defined by the relation $\Delta U = -W$. In terms of the potential energy, the work-energy theorem ($W = \Delta K$) is $-\Delta U = \Delta K$ or $\Delta K + \Delta U = 0$ or $\Delta(K+U) = 0$. The **mechanical energy** E is defined to be the sum of the kinetic and potential energies:

$$E \equiv K + U .$$

In terms of E, the work-energy theorem $\Delta(K+U) = 0$ is $\Delta E = 0$ and thus

$$E_{\text{inital}} = E_{\text{final}} .$$

This is the celebrated principle of **Energy Conservation**. During any mechanical process (*state i* $\rightarrow$ *state f*) involving conservative forces, the energy does not change ($E_i = E_f$). This is a remarkable feature of the motion. During the motion, *everything* (x , v , K , U) is changing in time – *except* for E! The energy can transform between K and U, but the total K+U must remain constant. Energy cannot be created or destroyed.

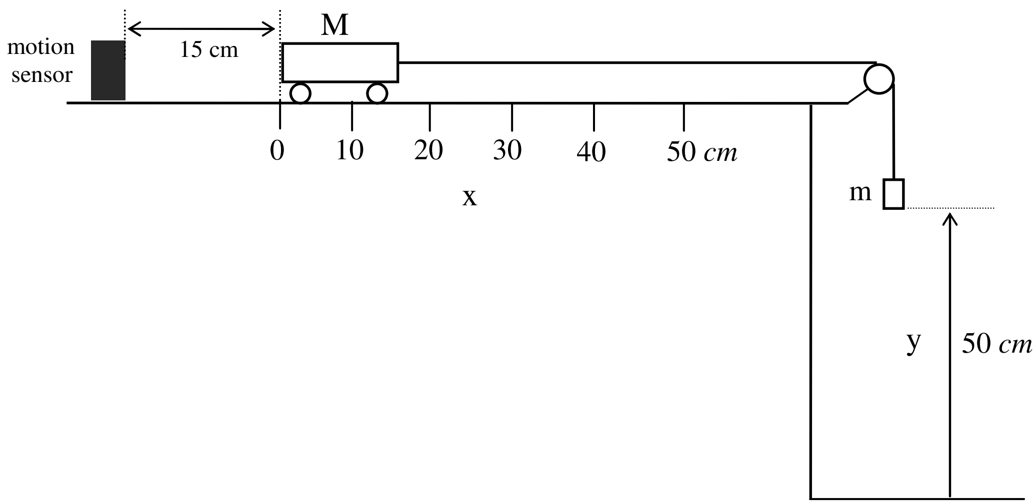
Introducing a quantity that <u>*does not change in time*</u> is the essence of all <u>*Conservation Laws*</u> in physics. Searching for conserved quantities in the universe is big business among professional physicists. In addition to conservation of *energy*, there is conservation of *momentum*, conservation of *charge*, and conservation of *mass*. In the theory of *relativity*, mass and energy are equivalent: $E = mc^2$. The motion of a mechanical system can be very chaotic with many system variables changing in time. Conservation Laws bring an *"order to the chaos"*.

<u>*Energy Exercise*</u>: A mass m = 2 is in free fall. The acceleration of gravity is g = 10. All numbers are in *mks* units. The position y(t) and velocity v(t) of the mass as a function of time t are $y(t) = 9 - 5t^2$ and $v(t) = -10t$. Find each of the energies (K , U , E) as a function of time t. *Hint*: substitute v(t) into K(v), substitute y(t) into U(y).

K = ______________ .   U = ________________ .   E = _______________ .

The System

You will study energy conservation in a system consisting of a cart of mass M connected via a string to a mass m = 30 *gram* (20*g*+10*g* weights) hanging over a pulley. Here is a schematic of the system in the initial state:



The Motion

For this experiment, it is crucial to have a level track. So once again, make sure the track is level using the "steel ball test". Open Logger Pro file *Changing Velocity 1*. Change the time scale on the graph to go from 0.0 to 3.0 *seconds*. Hold the system in the initial state – where the hanging mass is at y = 50 cm – and <u>*zero the motion sensor*</u> (click *Experiment* and select *Zero*). This sets the initial position of the cart to be x = 0 (see picture).

Starting in this initial resting state, click *Collect* and then release the cart. Let the sensor record the motion of the cart.

PRINT the whole screen showing the graph and the table of Time (t), Distance (x), Velocity (v) values recorded by the motion sensor.

In your table, identify four different x values that lie between x = 0.10 *m* and x = 0.40 *m* and are roughly equally spaced. Look for four values that are close to 0.10 , 0.20 , 0.30 , 0.40.
Of course your sensor will not capture these four whole numbers exactly but instead you might get something like 0.11 , 0.19 , 0.32 , 0.43.

<u>*Underline*</u> the four rows of data in the table that show the x and v values of the cart at the four different times. Record these values of x and v in the *Motion Table* below. You do not need to record the numerical values of the four times.

Motion Table

time	x (*m*)	v (*m/s*)
t_0	0	0
t_1		
t_2		
t_3		
t_4		

The Energy

Find the kinetic energy K, the potential energy U, and the mechanical energy E of your system (cart plus hanging mass) at the five times t_0 to t_4. Since K depends on velocity and U depends on position, you can calculate K and U from your five measured values of v and x.

<u>Hints</u>: K of the *system* is K(*cart*) + K(*hanging mass*), but U of the system is U(*hanging mass*). U(*cart*) = 0 since the cart does not change its vertical height. Let the potential energy U(y) of the system be zero when the hanging mass is on the ground (y = 0). You can find the five y-values of your hanging mass – needed to compute U(y) – from your five x-values listed in the *Motion Table* above. Note how x and y are related: when the cart moves from x = 0 to x = 10, the hanging mass falls from y = 50 to y = 40 (see picture above).

As a "Certified Energy Accountant" (CEA), fill in the following "Energy Bookkeeping Table":

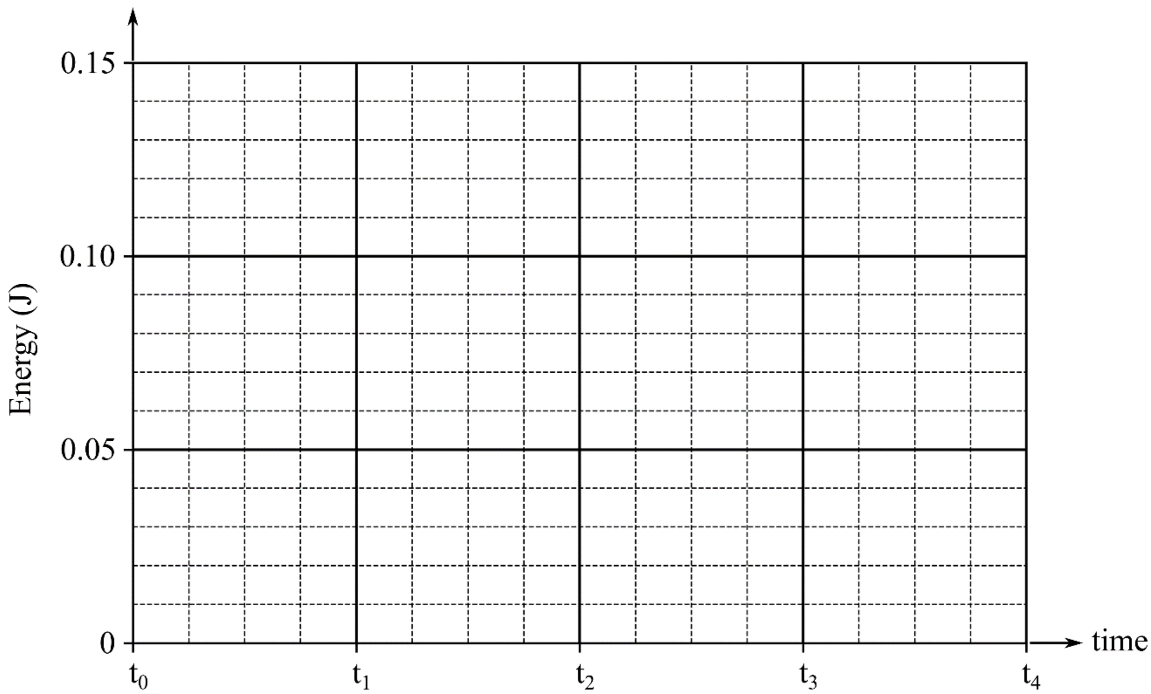
Energy Table

time	K (J)	U (J)	E (J)
t_0			
t_1			
t_2			
t_3			
t_4			

Provide the details of your calculation of K, U, and E at time t_3 in the space below:

Energy Analysis

What is happening to K, U and E as time passes? Using your data from the previous page, make a quantitative plot of these three energies in the graph below. Clearly label which curve represents K, U, and E. After doing this, make a fourth plot **in a different color** showing the theoretical value for E according to the **C.O.E. Law**. Label this 4th curve $E_{theoretical}$.



Complete the following sentence:

As time went from t_0 to t_4, the magnitude of K ___________________________ , the magnitude of U _____________________________________ , and the magnitude of the total mechanical energy, E, ____ .

Between t_1 and t_2, ΔK = ___________________ J and ΔU = _______________________ J.

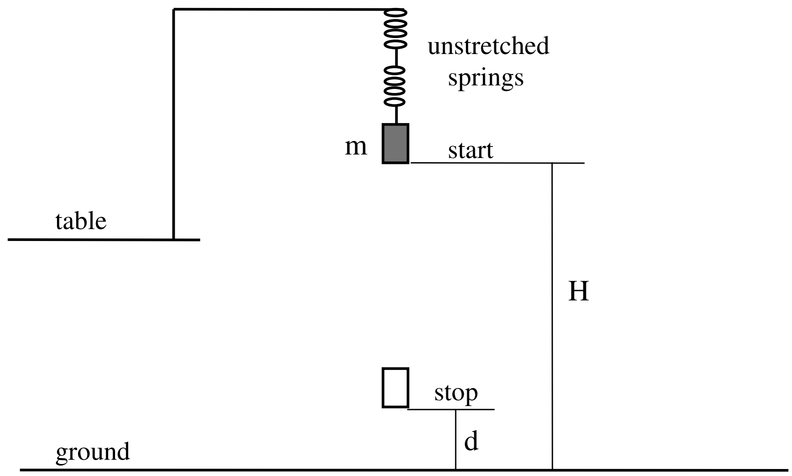
Explain what you think this means:

How much mechanical energy was lost to non-conservative forces from t_0 to t_4?

__________________________ milli-Joules

Part III. Designing a Bungee Jump System

A schematic of the system appears below. A 200 *gram* mass (the "person") hangs from two narrow silver springs in series (the "bungee cord").



Your design goal is to find the starting height H above the ground (floor) so that the mass, when released from rest, falls downward and stops at a point that is $d = 5.0 \pm 3.0$ *cm* above the ground, before being pulled back up by the stretched springs. Note that the "tolerance zone" for the final stopping height is 2 *cm* $< d < 8$ *cm*. Also note that in the starting position, the spring is in its relaxed (unstretched) state.

The Theory

System parameters:

- m = mass of "person".
- k = spring constant of "bungee cord."
- H = starting height above the ground.
- d = distance above the ground where the person momentarily stops.
- g = acceleration due to gravity near the surface of the earth.

In the space below, use the *Principle of Energy Conservation* to derive the theoretical formula that gives H as a function of m, k, d, g. Work with the symbols only. No numbers allowed. *Hint*: In the initial state, all the energy of the system is stored as gravitational potential energy (mass is at height H). In the final state, the energy is partly gravitational (mass is at height d) and partly elastic (spring is stretched a distance $H - d$). Note: If you are solving a quadratic equation, then you are solving the problem the hard way.

H = ______________________________ .

The Experiment

1. *Measure k*

Measure the value of k that characterizes the elasticity or "springiness" of your bungee cord. It is perfectly valid to treat the two-spring system as one spring whose force constant is k. To find k, hang a 100-gram weight from the bungee cord and measure the distance Δs the cord stretches. Compute the force-per-stretch ratio, $k = F/\Delta s$. Repeat with a 200-gram mass. Average your two k values. Show your data and calculation of k in the space below.

k (average) = ______________ *N/m* .

Calculate the predicted value of H by inserting your measured value of k and the known values of m, d, and g into your theoretical formula for H. Show your calculation.

Predicted H = _______________ *m* .

2. *Perform the Bungee Jump*

Hook the m = 200 *gram* mass to the end of the spring. Make sure that the spring is in its relaxed (unstretched) state. This is the initial condition. Raise or lower the horizontal rod from which the spring hangs so that the bottom of the mass m is at the predicted height H above the floor (see schematic). You are now ready to release the mass. Observe the bungee jump and hope that the mass momentarily stops before hitting the ground.

3. *Results of the Jump*

If your bungee jump was successful, how far from the ground was the mass when it momentarily came to rest? To measure d, stack enough wooden blocks on the floor directly below the falling mass so that the mass barely taps/misses the top of the blocks.

actual d = ________________ *cm* .

Is your actual d within the tolerance zone (2.0 *cm* $\rightarrow$ 8.0 *cm*) of the *design specs* ? What are the sources of errors in *The Experiment*, and the approximations that you made in *The Theory*, that could account for any discrepancy?