Team: ________________

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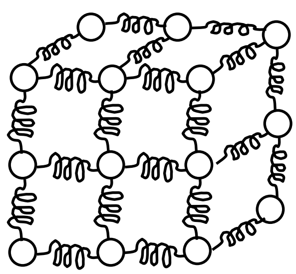
Newton's Third Law, Momentum, Center of Mass

Part I. Newton's Third Law

Atomic Springs

When you push against a wall, you feel a force in the opposite direction. The harder you push, the harder the wall pushes back on you. The amazing fact is that the force exerted by the wall on you is *exactly equal and opposite* to your push. How does the wall "know" to push this way? In general how can inanimate objects, such as walls, floors, and tables, push and pull other objects?

The answer lies in the atomic world. Solids are made of atoms held together in a lattice structure by electric forces (atomic bonds). Physicists view solids as a set of balls (atoms) connected by springs (bonds):

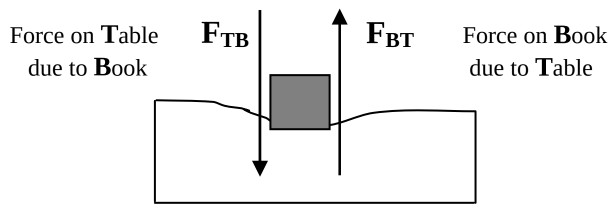


The length of one "atomic spring" is about one *nanometer* (10^{-9} m). The number of atoms in a typical solid such as a book or a table is about 1,000,000,000,000,000,000,000,000 !

When you push on a solid, the atomic springs compress, i.e., the electron clouds of the atoms overlap. The harder you push, the more the springs compress, and thus the harder the springs push back on you. What you actually "feel" is the *electric repulsion* between the atoms in the surface of the solid and the tip of your finger. On a larger scale, it is similar to the *magnetic repulsion* you feel when you push one magnet toward another magnet.

It is important to realize that nothing actually "touches" when two objects come into "contact", i.e., when you push on something. If two atoms actually touched – one nucleus on top of another nucleus – then nuclear fusion would result thereby creating an atom-bomb explosion! Force *fields* touch, not ponderable *matter*. Again, think about two magnets repelling at a distance.

This "atomic springiness" endows matter with an elastic property. When you put a book on the table or push on the wall, the surface of the table and the wall bend slightly (greatly exaggerated in the picture below). Think of putting a bowling ball on a mattress (box springs).



The "bent" table acts like a compressed spring that exerts a force back on the book. The "*action*" of pushing forward creates a "*reaction*" of pushing backward.

Not all solids are *elastic*. Materials such as clay, putty, and dough, are *inelastic* – they *do not "spring back"* after they have been compressed or stretched.

Action and Reaction

Qualitatively speaking, a force is a 'push' or a 'pull'. Rigorously speaking, *a force is an interaction between two objects.* This interaction obeys a deep law of mechanics:

Newton's Third Law: If one object exerts a force on a second object, then the second object exerts an equal and opposite force on the first. In symbols,

$$\mathbf{F}_{12} = -\mathbf{F}_{21} \ .$$

Notation: $\mathbf{F}_{12} \equiv$ Force *on* object 1 *due* to object 2. $\mathbf{F}_{12} \equiv$ Force *on* object 2 *due* to object 1.

All forces come in pairs. For every "*action*", there is an equal and opposite "*reaction*". You cannot have a single force in any situation because the force on *one object* is always due to some *other object*. Here are some examples of interactions (pairs of forces):

Action	Reaction	
You push back on **F**loor	**F**loor pushes forward on **Y**ou	(explains how a person walks)
Tires push back on **R**oad	**R**oad pushes forward on **T**ires	(explains how a car moves)
Rocket pushes back on **G**as	**G**as pushes forward on **R**ocket	(explains how a jet moves)
Book pushes down on **T**able	**T**able pushes up on **B**ook	
Earth pulls down on **M**oon	**M**oon pulls up on **E**arth	
Magnet pulls left on **N**ail	**N**ail pulls right on **M**agnet	

Add three more force pairs to this list:

____________________________ ____________________________

____________________________ ____________________________

____________________________ ____________________________

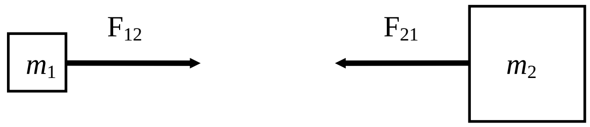
Experimental Tests of Newton's Third Law

The Perfect Balance

Since *F* is equal to *ma*, the equal and opposite nature of the <u>force</u> *F* implies an equal and opposite nature in the <u>motion</u> *ma*. Any pair of mutually-interacting objects (isolated system) must move toward or away from each other in such a way that their *ma*'s are in <u>perfect balance</u> at all times. The *symmetry in the motion* "$\boldsymbol{m_1 a_1 = - m_2 a_2}$" is a reflection of the *symmetry in the force* "$\mathbf{F_{12} = - F_{21}}$" .

Here you will study three different kinds of *interaction* between a pair of carts: <u>spring force</u> , <u>magnetic force</u> , and <u>contact force</u>. You will observe first hand how the motions of each cart are in "perfect balance". Verifying this symmetry in the motion is the gold standard test of *Newton's Third Law*.

Here is the force diagram of the interacting carts:



Here is the *Physics* of the motion:

During the time interval Δt during which the carts interact,

F_{12} causes the velocity of m_1 to change by an amount Δv_1 .
F_{21} causes the velocity of m_2 to change by an amount Δv_2 .

Newton's Second Law of Motion applied to each cart separately gives

$$F_{12} = m_1 \Delta v_1/\Delta t \qquad\qquad F_{21} = m_2 \Delta v_2/\Delta t \ .$$

These second-law equations allow you to find the average *forces* F_{12} and F_{21} during the *interaction time* Δt by observing the *change in velocities* Δv_1 and Δv_2 .

Qualitative Experiments on Newton III

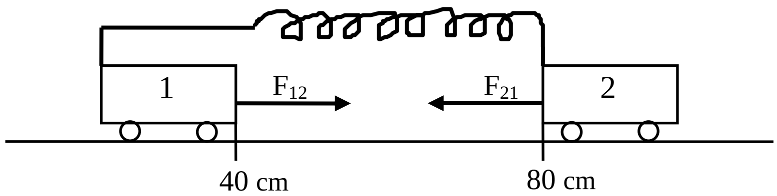
Here, you will deduce the values of F_{12} and F_{21} from *qualitative estimates* of Δv_1 and Δv_2 . You do not have to quantitatively measure any velocity. Simply observe the motion and note if the velocities look the "same". If you prefer more accuracy, you can do a quick measurement using the meter scale and a stopwatch. When observing the motion, here is the "bottom line":

Look for the "*symmetry*", the "*perfect balance*", or the "*gain = loss*" in the motion.

Experiment 1. Interaction = Spring Force

Observe the Motion

First make sure that the *track is level* using the leveler. Attach a spring between the carts as shown below. Place cart 1 at the 40-cm mark and cart 2 at the 80-cm mark. The non-magnetic Velcro patches should be facing each other so that the carts stick together when they collide. Release the carts (simultaneously).



Observe the motion. Note that it takes about one second of time from the moment of release to the moment of collision. During this *interaction time*, the spring force causes the velocity of each cart to change.

Given the following fact about the velocity of cart 1, describe the velocity of cart 2 (fill in the blanks) based on your qualitative observations. Define the positive direction to point to the right.

During the interaction time 1.0 *second*,

The velocity of cart 1 changes from 0 m/s to 0.30 m/s .

The velocity of cart 2 changes from _____________ to _____________ .

Find the Action and the Reaction

Measure the mass of each cart. Record the *change in velocity* of each cart during the *interaction time* Δt = 1.0 *s*.

m_1 = _____________ *kg* . m_2 = _____________ *kg* .

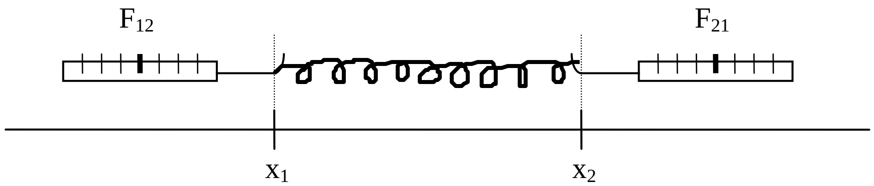
Δv_1 = _____________ *m/s* . Δv_2 = _____________ *m/s* .

Calculate the average values of the *spring force* acting on each cart during the interaction.

F_{12} = ________________ *N* . F_{21} = ________________ *N* .

Measuring F_{12} and F_{21} with Spring Scales

Make sure that both scales read "zero" when held in the horizontal position and nothing is attached to the hooks. Remove the spring that connects the carts. Attach the two scales to the two ends of the spring:



Pull the scales to various positions (x_1 , x_2) along the track listed in the table below and record the scale readings (*magnitude* and *direction*) in the table. Remember, right is + and left is – .

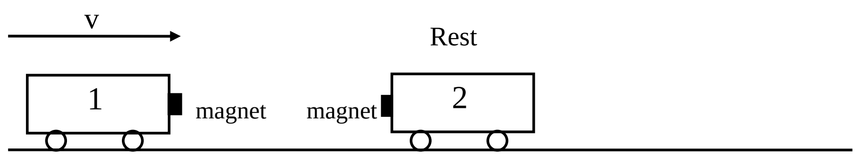
x_1 (*cm*)	x_2 (*cm*)	F_{12} (*N*)	F_{21} (*N*)
40	80		
45	75		
50	70		
55	65		

What is your conclusion?

Experiment 2. Interaction = Magnetic Force

Observe the Motion

Place cart 1 at the left end of the track. Place cart 2 in the middle of the track. Make sure that the cart magnets are facing each other so that the carts repel each other. Give cart 1 an initial velocity of about 30 *cm/s* toward cart 2.



Observe the motion. Note that the *interaction time* is about 0.5 *seconds*. This is the time interval during which the carts are close enough (< 10 cm) to feel the magnetic force and repel each other. During this time, the magnetic force causes the velocity of each cart to change: cart 1 slows down, cart 2 speeds up. Before and after this time of interaction (when the carts are separated by more than 10 cm) the net force on each cart is approximately zero and thus the carts move at constant velocity. Friction does cause some slowing.

Given the following fact about the velocity of cart 1, describe the velocity of cart 2 (fill in the blanks) based on your qualitative observations.

During the interaction time 0.50 *seconds*,

The velocity of cart 1 changes from 0.30 m/s to 0 m/s .

The velocity of cart 2 changes from ____________ to ____________ .

Find the Action and Reaction

Measure the mass of each cart. Record the *change in velocity* of each cart during the *interaction time* Δt = 0.50 *s*.

m_1 = ____________ *kg* . m_2 = ____________ *kg* .

Δv_1 = ____________ *m/s* . Δv_2 = ____________ *m/s* .

Calculate the average values of the *magnetic force* acting on each cart during the interaction.

F_{12} = _______________ *N* . F_{21} = _______________ *N* .

Experiment 3.   Interaction = Contact Force (Electron Cloud Repulsion)

Observe the Motion

Place cart 1 at the left end of the track. Place cart 2 at the center. Make sure that the non-magnetic Velcro patches (hooks – loops) are facing each other so that the carts stick together after the collision. Give cart 1 an initial velocity of about 30 *cm/s* toward cart 2.

Observe the motion. The *interaction time* is about 0.5 *seconds*. During this time of "contact", the contact force causes the velocity of each cart to change: cart 1 slows down, cart 2 speeds up. Remember that nothing actually touches. During the contact – the "smacking" of the carts – the distance between the electron clouds in 1 and 2 is a few nanometers!

Given the following fact about the velocity of cart 1, describe the velocity of cart 2 by filling in the blanks.

During the interaction time 0.5 *seconds*,

The velocity of cart 1 changes from   0.30 m/s   to   0.15 m/s   .

The velocity of cart 2 changes from   _____________   to   _____________ .

Find the Action and the Reaction

Measure the mass of each cart. Record the *change in velocity* of each cart during the *interaction time* Δt = 0.5 *s*.

m_1 = _____________ *kg* . m_2 = _____________ *kg* .

Δv_1 = _____________ *m/s* . Δv_2 = _____________ *m/s* .

Calculate the average values of the *contact force* (repelling electron clouds) acting on each cart during the interaction.

F_{12} = ________________ *N* . F_{21} = ________________ *N* .

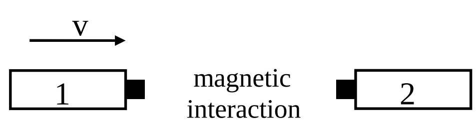
Quantitative Experiments on Newton III

Here you will use *two motion sensors* interfaced to two computers. Sensor 1 will measure the velocity and acceleration of cart 1. Sensor 2 will measure the velocity and acceleration of cart 2. Place the sensors on the table off each end of the track.

Open Logger Pro file *Changing Velocity 1*. Change the setting on Sensor 2 so that it sees objects that are moving to the right as moving in the positive direction: On the computer that Sensor 2 is connected to, click on "Setup" (on menu bar), then "Sensors", then "Details", then "Reverse Direction". Now change the graph features as follows. The graph window displays plots of **x** and **v**. Change this to **v** and **a** (by clicking on the **x** and **v** symbols that label each vertical axis). Change the **t** scale to go from 0.0 to 3.0 *s*. Change the **v** scale to go from – 0.50 to 0.50 *m/s*. Change the **a** scale to go from – 2.0 to 2.0 m/s^2 . You are now ready to record the motion.

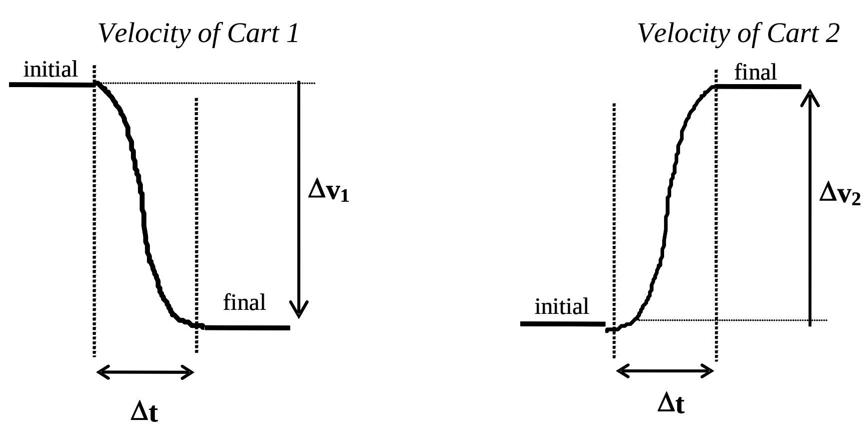
Experiment 1. Magnetic Interaction (Equal Mass Carts)

Give cart 1 an initial velocity toward cart 2 (initially at rest):



Velocity Curves

Record the motion. Your velocity curves (v as a *function of time*) should have the following overall appearance:



Each velocity curve is divided into three distinct regions:

1. <u>Constant Velocity</u> *before* the interaction (initial).
2. <u>Changing Velocity</u> *during* the interaction (time interval Δt).
3. <u>Constant Velocity</u> *after* the interaction (final).

Note how the velocity curves clearly display the three motion parameters (Δt , Δv_1 , Δv_2) that determine the force of interaction. If your **v** graphs do not look like the above **v** graphs, then you may have to adjust the location and/or tilt of your motion sensors. You can also try different initial positions of cart 2.

Print your graphs. Write the numerical values of the *initial* velocity, the *final* velocity, and the *change* in velocity of each cart directly on the corresponding parts of your velocity curves.

Momentum Balance

From your measured values of the *velocity change*, calculate the *momentum change* of each cart during the interaction.

Momentum Lost by Cart 1:

$-m_1\Delta v_1$ = – (*kg*) (*m/s*) = ______________ *kgm/s*.

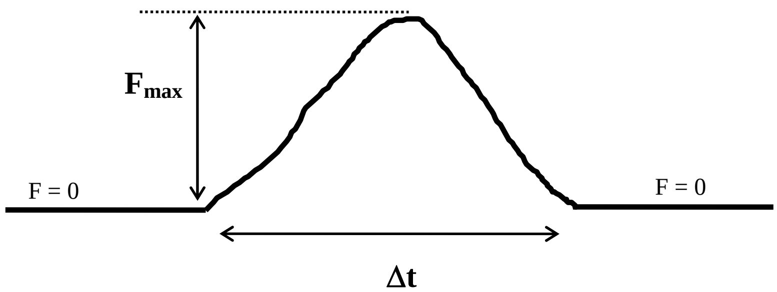
Momentum Gained by Cart 2:

$m_2\Delta v_2$ = (*kg*) (*m/s*) = ______________ *kgm/s*.

Does the momentum in you system seem to be "*conserved* "? Explain.

Force Curves

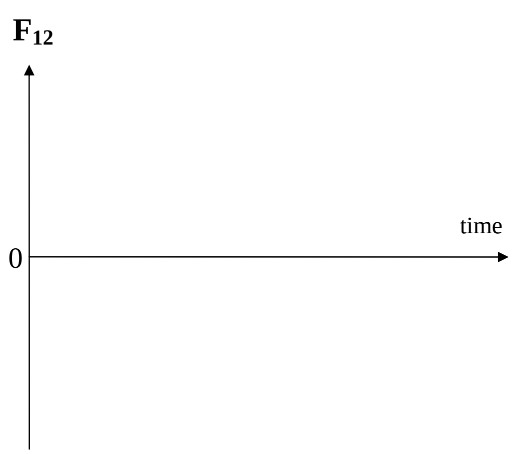
During the interaction, the magnetic force on each cart varies with time. This time-dependent force function starts at zero (when the carts are far apart), then rapidly rises (as the carts get closer), then reaches a maximum (when the carts almost touch), and then falls back to zero (as the carts move farther apart). The force on cart 2 as a <u>*function of time*</u> has the following overall shape:

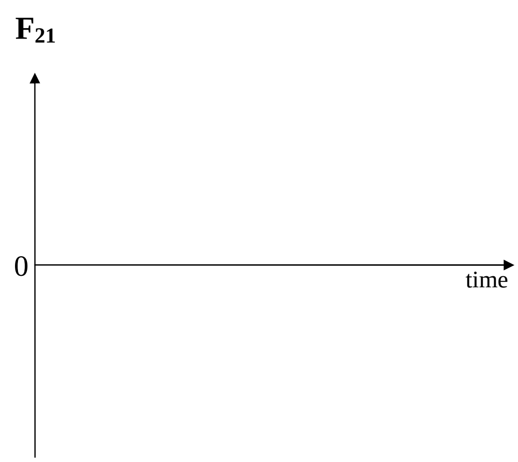


Note that the force curve is characterized by two parameters (height and width): the ***maximum force*** F_{max} and the *interaction time* Δt. The force F_{max} occurs when the carts are closest to each other with their magnets repelling most strongly. As always, Δt is the time interval during which one cart feels the force due to the other cart. The *average value* of the time-dependent force (over the time Δt) is roughly $\frac{1}{2}F_{max}$.

What is the exact shape of this *interaction curve*? Force curves and power curves are very important in science and engineering. Since **F** is equal to **ma**, you can find the **F** *curve* by measuring the **a** *curve*. Look at the **a** *curves* recorded by your motion sensors. The F_{12} curve is equal to the a_1 curve multiplied by m_1 . The F_{21} curve is equal to the a_2 curve multiplied by m_2. This multiplication by mass simply changes the vertical scale of your graph. Show this multiplication directly on your printed graphs of acceleration.

Sketch your measured force curves below. Write the numerical values of the height (*maximum force*) and width (*interaction time*) on each curve.





Conclusion:

Experiment 2. Magnetic Interaction (Unequal Masses)

Give mass 1 (*one* cart) an initial velocity toward mass 2 (*two* stacked carts, initially at rest):

Record the motion. Make sure that your graphs clearly display the before, the during, and the after parts of the motion. You may have to adjust the motion sensor and/or the initial location of mass 2.

Print your graphs. Write the numerical values of the *initial* velocity, the *final* velocity, and the *change* in velocity of each mass directly on the corresponding parts of your velocity curves.

<u>Momentum Balance</u>

From your measured values of the *velocity change*, calculate the *momentum change* of each mass during the interaction.

Momentum Lost by Mass 1:

$- m_1 \Delta v_1$ = – (*kg*) (*m/s*) = ______________ *kgm/s*.

Momentum Gained by Mass 2:

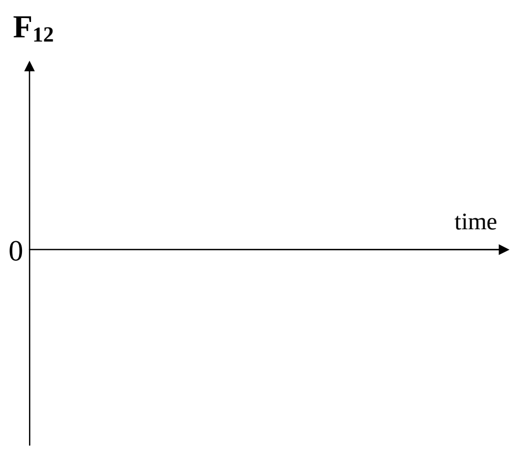
$m_2 \Delta v_2$ = (*kg*) (*m/s*) = ______________ *kgm/s*.

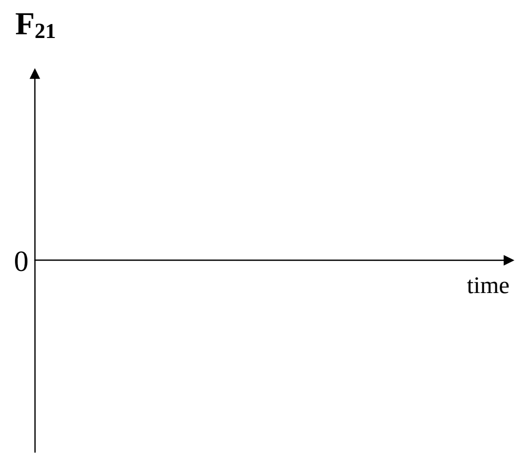
Is the *velocity* lost by 1 equal to the *velocity* gained by 2? Does the *momentum* in you system seem to be "*conserved*"? Explain.

Force Curves

Find the forces acting on m_1 and m_2 as ***functions of time***. The F_{12} curve is equal to the a_1 curve multiplied by m_1 . The F_{21} curve is equal to the a_2 curve multiplied by m_2. Show this multiplication directly on your printed motion-sensor graphs of the acceleration. Be careful with this multiplication: m_1 is *not* equal to m_2 .

Sketch your measured force curves below. Write the numerical values of the height (*maximum force*) and width (*interaction time*) on each curve.





Conclusion:

Part II. Conservation of Momentum

What does "*action = reaction*" have to do with "*momentum = constant*"? Everything! From your experimental investigations into *Newton's Third Law*, you discovered that the "amount of mv" lost by one object is equal to the "amount of mv" gained by the other object. This means that the "total amount of mv" in the system cannot be created or destroyed – it is conserved!

The *Physics* behind Momentum Conservation is simple. Just combine Newton's Laws II and III: Write the *Force* law " $\mathbf{F_{12} = -F_{21}}$ " as the *Motion* law " $\mathbf{m_1 \Delta v_1 / \Delta t = -m_2 \Delta v_2 / \Delta t}$ ". Cancel the mutual time Δt of interaction, and there you have it, one of the most celebrated principles in science:

$$\Delta(m_1 v_1) \quad = \quad -\Delta(m_2 v_2)$$

Momentum Gained by Object 1 = *Momentum Lost by Object 2*

This "*gain = loss*" relation means that the **<u>Total Momentum</u>** $m_1v_1 + m_2v_2$ of the isolated system <u>cannot change</u>:

$$\Delta(m_1 v_1 + m_2 v_2) = 0 \quad \text{implies} \quad m_1 v_1 + m_2 v_2 = \textit{constant} .$$

Our derivation of "$m_1v_1 + m_2v_2 = \textit{constant}$ " is valid for any kind of *isolated system* – a system for which the *net external force is zero*. The system could be two billiard balls, two colliding galaxies, or the system of 10^{28} air molecules in our lab room. Physicists write the general *Principle of Momentum Conservation* simply as **<u>P</u>** <u>= *constant*</u>.

A Set of Collision Experiments

You will study different collisions between your two carts. During each collision (interaction), the carts exert strong internal forces on each other. Remember that the internal forces (the force of mutual interaction) cannot change the total momentum of the system (since $F_{12} + F_{12} = 0$). If you neglect the external force of friction (since it is much weaker than the internal force), then you can conclude that the total momentum before the collision must equal the total momentum after the collision:

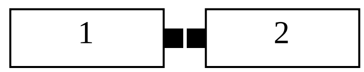
$$[\, m_1 v_1 + m_2 v_2 \,]_{initial} = [\, m_1 v_1 + m_2 v_2 \,]_{final} \; .$$

$$[\, p_1 + p_2 \,]_{initial} = [\, p_1 + p_2 \,]_{final}$$

This simple equation tells you a whole lot about the before and after motion in the collision process – *without having to know anything about the complicated internal collision forces (springs, magnets, electrons, velcro, etc.) that are causing the motion*!

Experiment 1. Fly Apart Reaction: Magnetic Explosion

Here is the initial state of the system:



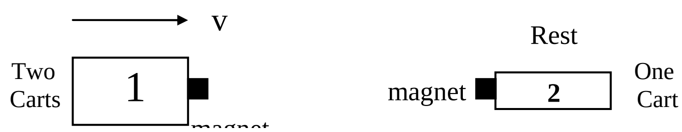
Push the carts together with your hands until the carts "touch". In this initial state, the magnetic repulsion is ready to "explode" the system. Release both carts at the same time and observe them "fly apart". Based solely on your observation (no quantitative measurements), sketch three momentum curves (*momentum as a function of time*): p_1 of cart 1 , p_2 of cart 2 , and p_1+p_2 of the system. Label your three curves.

Experiment 2. Velocity Exchange Reaction

Sketch the three momentum curves for the following collision. Here is the initial state:

Experiment 3. Big Mass Colliding with Small Mass

Here is the initial motion:



Run the experiment. Make the initial v somewhere between 30 and 50 *cm/s*. Use *two motion sensors* (located off each end of the track) to record the velocity. Write your initial and final velocities on the pictures. Fill in the *Momentum Table*.

Initial Velocities *Final Velocities*

____________ *m/s* ___0 _ *m/s* ____________ *m/s* ____________ *m/s*

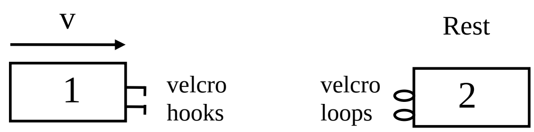
Momentum Table

	Initial	*Final*
p_1		
p_2		
$p_1 + p_2$		

How well (percent difference) do your results provide an experimental proof of *the Law of Momentum Conservation*?

Experiment 4.   Totally Inelastic (Sticking) Collision

Here is the initial state of the system:



Run the experiment. Make the initial v somewhere between 30 and 50 *cm/s*. Use *one motion sensor* (located off the left end of the track) to record the velocity. Write your initial and final velocities on the pictures.   Fill in the *Momentum Table*.

Initial Velocity   *Final Velocity*

___________ *m/s*   __0__ *m/s*   ___________ *m/s*

Momentum Table

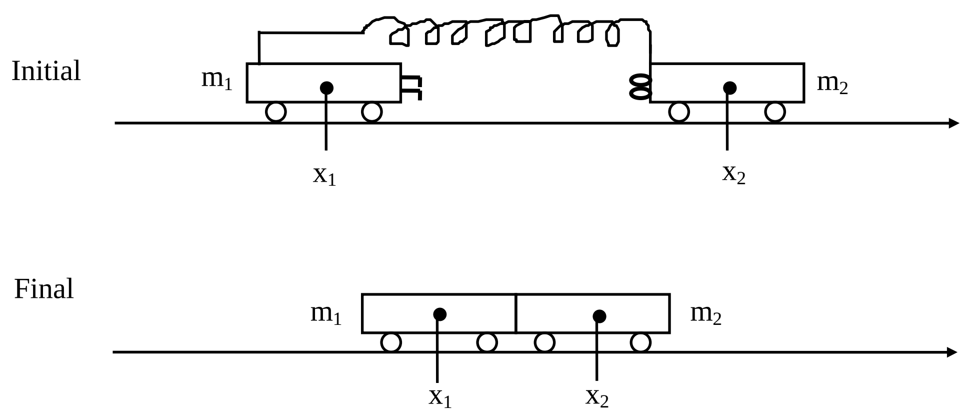
	Initial	*Final*
p_1		
p_2		
$p_1 + p_2$		

How well (percent difference) do your results provide an experimental proof of *the Law of Momentum Conservation*?

Part III. Conservation of The Center of Mass

The Experiment: Where do the Carts Meet ?

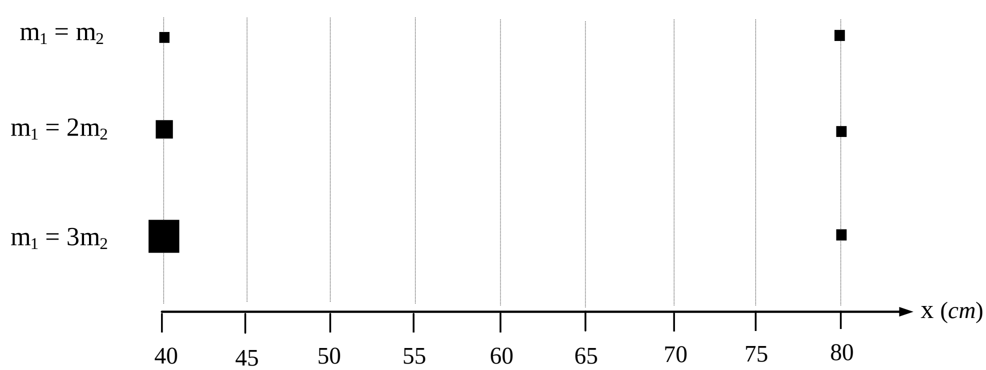
Attach the spring between the carts. Pull the carts apart so that the *center* of cart 1 is at $x_1 = 40$ *cm* and the *center* of cart 2 is at $x_2 = 80$ *cm*. The x-axis is defined by the meter scale fixed along the track. The non-magnetic Velcro patches should be facing each other so that the carts stick together when they collide. Release the carts at the same time. Where do the carts meet?



Perform this experiment using three different pairs of masses: $m_1 = m_2$, $m_1 = 2m_2$, $m_1 = 3m_2$. Stack carts to achieve these multiple mass ratios. Start the system in the same initial state ($x_1 = 40$ *cm* , $x_2 = 80$ *cm*) each time. Record your measured values of the position $x_1(f)$ of cart 1 and the position $x_2(f)$ of cart 2 in the final state:

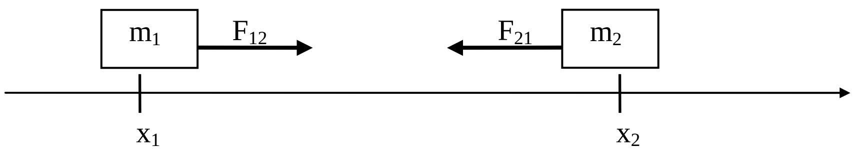
m_1 (kg)	m_2 (kg)	$x_1(f)$ (cm)	$x_2(f)$ (cm)

The initial positions (center points) of cart 1 and cart 2 are shown on the following diagram. Mark the final positions that you measured.



The Center of Mass is the Place to Be

To understand the physics of this experiment, it is most revealing to analyze the motion in terms of the "center of mass". You have seen how the quantity "$m_1v_1+m_2v_2$" is conserved. Here you will discover how the quantity "$m_1x_1+m_2x_2$" is conserved. As always, start with free body diagrams and Newton's Second Law to get a handle on the forces and the motions. Neglect friction (since it is much smaller than the spring force).



$F_{12} = m_1\, d^2x_1/dt^2 \qquad\qquad F_{21} = m_2\, d^2x_2/dt^2$

Adding these relations gives the motion equation: $\mathbf{F_{12}} + \mathbf{F_{21}} = d^2(\mathbf{m_1x_1 + m_2x_2})/dt^2$.

Since the quantity "$m_1x_1 + m_2x_2$" naturally appears on the right hand side of Newton's motion equation, it must be an important quantity! Indeed, this "mass-times-position" quantity underlies the basic concept of the "center of mass". Here is the formal definition:

The *<u>Center of Mass</u>* x_{cm} of a system of two particles is defined by the relation

$$x_{cm} \equiv \frac{m_1\, x_1 + m_2\, x_2}{m_1 + m_2} .$$

In terms of $a_{cm} \equiv d^2x_{cm}/dt^2$, the motion equation becomes: $\mathbf{F_{12}} + \mathbf{F_{21}} = (\mathbf{m_1+m_2})\, \mathbf{a_{cm}}$.

Newton's Third Law $\mathbf{F_{12}} = -\mathbf{F_{21}}$ reduces the motion equation to: $\mathbf{a_{cm}} = \mathbf{0}$.

The Conservation of x_{cm}

What could be simpler! The acceleration of the *center of mass* of the system is equal to zero. In contrast, the accelerations of the *individual parts* (mass 1 and mass 2) of the system are nonzero and far from simple. F_{12} (F_{21}) causes m_1 (m_2) to accelerate in a complicated time-dependent way. Amidst this *chaos* of accelerating parts, there is one point of *calmness*: the center of mass.

In summary, the internal (spring) forces are definitely responsible for the motion of each cart, but here is the amazing thing: *these same internal forces have absolutely no effect on the motion of the center of mass* ... all because the net internal force $F_{12} + F_{21}$ is <u>exactly equal to zero</u>.

So what exactly is x_{cm} doing all the while the carts are moving and their positions x_1 and x_2 are rapidly changing? The center of mass does not move at all !! Initially, when the two carts are pulled apart, the center of mass is located at some point x_{cm} (*initial*) between the carts. Since this point cannot accelerate ($a_{cm} = 0$), and since it is initially rest, it must ***remain at rest for all future time***. The center of mass is a *constant of the motion*:

$$x_{cm}\text{ (initial)} = x_{cm}\text{ (final)} .$$

This Theorem on "*The Conservation of x_{cm}*" theorem tells you a whole lot about the motion of the system – without having to know anything about the complicated internal forces (springs, magnets, electrons, velcro, etc.) within the system that are causing the motion!

Einstein, Center of Mass, $E = mc^2$

Albert Einstein realized the deep significance of the constancy of x_{cm}. In his 1906 paper, *"The Principle of Conservation of Motion of the Center of Mass and the Inertia of Energy"*, Einstein used the relation x_{cm} (initial) = x_{cm} (final) to derive the most famous equation in science: $\mathbf{E = mc^2}$. Einstein used this center of mass theorem to analyze the motion of a train car on a frictionless track when a light bulb inside the car was turned on. In this lab, you will use the same theorem to analyze the motion of two carts on a frictionless track when the spring between the carts is released.

Calculating the Center of Mass

Calculate x_{cm} (initial) from the initial values of x_1 and x_2. Calculate x_{cm} (final) from your final measured values of x_1 and x_2 (displayed in the previous table). Show your calculations. Record your values in the *Center of Mass Table*.

Center of Mass Table

m_1	m_2	x_1(i)	x_2(i)	x_1(f)	x_2(f)	x_{cm}(i)	x_{cm}(f)
		40	80				
		40	80				
		40	80				

Based on your experimental results, is the center of mass of your system a *constant of the motion* (to within 10%)? Explain.