Team: ________________

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Newton's Third Law, Momentum, Center of Mass

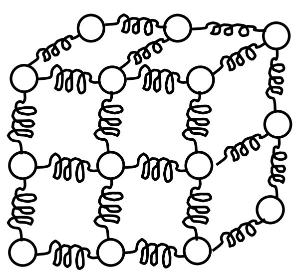
Newton's Third Law is a deep statement on the *"symmetry of interaction"* between any two bodies in the universe. How is the pull of the earth on the moon related to the pull of the moon on the earth? How is the impact of a bat on a ball related to the impact of the ball on the bat? *Newton's Third Law* is intimately related to two huge principles in physics: *Conservation of Momentum* and *Motion of the Center of Mass*.

Part I. Newton's Third Law

Atomic Springs

When you push against a wall, you feel a force in the opposite direction. The harder you push, the harder the wall pushes back on you. The amazing fact is that the force exerted by the wall on you is *exactly equal and opposite* to your push. How does the wall "know" to push this way? In general how can inanimate objects, such as walls, floors, and tables, push and pull other objects?

The answer lies in the atomic world. Solids are made of atoms held together in a lattice structure by electric forces (atomic bonds). Physicists view solids as a set of balls (atoms) connected by springs (bonds):

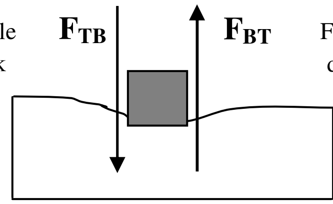


The length of one "atomic spring" is about one *nanometer* (10^{-9} m). The number of atoms in a typical solid such as a book or a table is about 1,000,000,000,000,000,000,000,000 .

When you push on a solid, the atomic springs compress, i.e., the electron clouds of the atoms overlap. The harder you push, the more the springs compress, and thus the harder the springs push back on you. What you actually "feel" is the <u>*electric repulsion*</u> between the atoms in the surface of the solid and the tip of your finger. It is similar to the *magnetic repulsion* you "feel" when you push one magnet toward another magnet.

It is important to realize that nothing actually "touches" when two objects come into "contact", i.e., when you push on the wall with your hand. If two atoms actually touched – one nucleus on top of another nucleus – then nuclear fusion would result thereby creating an atom-bomb explosion! Force *fields* touch, not ponderable *matter*.

This "atomic springiness" endows matter with an elastic property. When you put a book on the table or push on the wall, the surface of the table and the wall bend slightly (greatly exaggerated in the picture below). Think of putting a bowling ball on a mattress (box springs).



The "bent" table acts like a compressed spring that exerts a force back on the book. The *action* of pushing downward creates a *reaction* of pushing upward. This is the atomic origin of the ubiquitous *Normal Force*.

"The Law of Action/Reaction"

Qualitatively speaking, a force is a 'push' or a 'pull'. Rigorously speaking, *a force is an <u>interaction</u> between two objects*. This interaction obeys a deep law of mechanics:

<u>Newton's Third Law</u>: If one object exerts a force on a second object, then the second object exerts an equal and opposite force on the first. In symbols,

$$\mathbf{F_{12}} = -\mathbf{F_{21}}$$

$\mathbf{F_{12}} \equiv$ Force *on* object 1 *due* to object 2. $\mathbf{F_{21}} \equiv$ Force *on* object 2 *due* to object 1.

The minus sign in Newton's Third Law is one of the most important minus signs in all science. All forces come in pairs. For every "*action*", there is an equal and opposite "*reaction*". You cannot have a single force in any situation because the force on *one object* is always due to some *other object*. Here are some examples of interactions (pairs of forces):

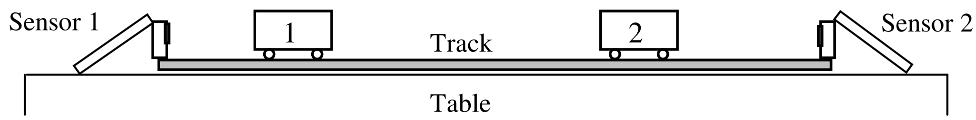
<u>Action</u>	<u>Reaction</u>
Foot push backward on **F**loor	**F**loor pushes forward on **F**oot (explains how a person moves)
Tire push backward on **R**oad	**R**oad pushes forward on **T**ire (explains how a car moves)
Rocket pushes back on **G**as	**G**as pushes forward on **R**ocket (explains how a jet moves)
Book pushes down on **T**able	**T**able pushes up on **B**ook
Earth pulls down on **M**oon	**M**oon pulls up on **E**arth
Magnet pulls left on **N**ail	**N**ail pulls right on **M**agnet

Add three more force pairs to this list:

____________________________	___________________________
____________________________	___________________________
____________________________	___________________________

Part II. Experimental Tests of Newton's Third Law

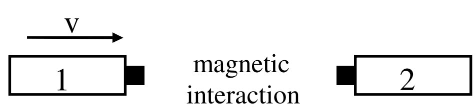
First make sure the track is level using the "steel ball test". Here you will use *two motion sensors* interfaced to two computers. Sensor 1 will measure the velocity and acceleration of cart 1. Sensor 2 will measure the velocity and acceleration of cart 2. Tilt open each sensor and place at each end of the track as shown below.



Open the program Logger Pro. Open the file "Sensor 1" on the computer connected to sensor 1. Open the file "Sensor 2" on the computer connected to sensor 2.

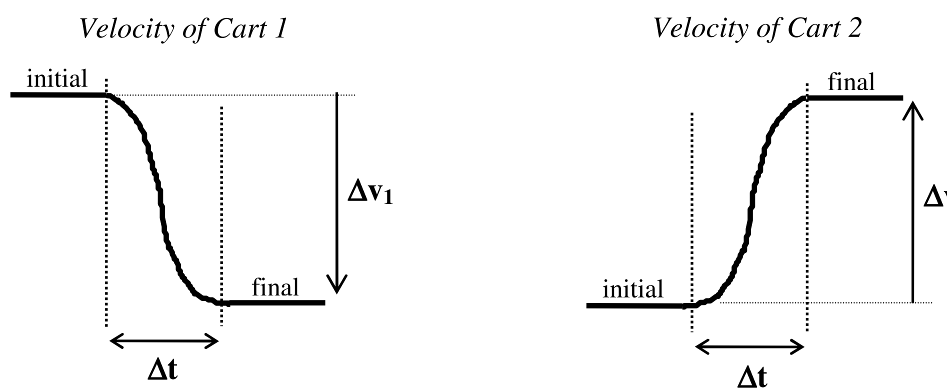
Experiment 1. Magnetic Interaction (Equal Mass Carts)

Make sure the cart magnets are facing each so that the action-reaction force between the carts is a force of repulsion. Give cart 1 an initial velocity (30 − 50 *cm/s*) toward cart 2 (initially at rest). Record the motion of each cart.



Velocity Curves

Your velocity curves (v as a *function of time*) should have the following overall appearance:



A velocity curve is divided into three distinct regions:

1. Constant Velocity (horizontal line) *just before* the interaction (initial).
2. Changing Velocity (steep sloping line) *during* the interaction (time interval Δt).
3. Constant Velocity (horizontal line) *just after* the interaction (final).

CHANGE Graph Scales: Adjust the t and v scales on your graphs so that your velocity curves resemble the pictures above. The initial and final velocity "plateaus" should clearly fill the whole screen. Do not display any "bad data" that is outside the three regions defined above. Note that the *interaction time* Δt is the interval of time during which each cart "feels" the force due to the other cart. During the interaction time Δt ,

F_{12} causes the velocity of mass m_1 to change by an amount Δv_1 .
F_{21} causes the velocity of mass m_2 to change by an amount Δv_2 .

PRINT your graphs. WRITE the numerical values of Δt , Δv_1 , Δv_2 on the corresponding parts of your velocity curves.

<u>Momentum</u> (gain / loss) & <u>Force</u> (action / reaction).

From your measured values of the *mass* and the *velocity change*, calculate the *momentum change* of each cart during the interaction.

$m_1\Delta v_1$ = (*kg*) (*m/s*) = ______________ *kgm/s*.

$m_2\Delta v_2$ = (*kg*) (*m/s*) = ______________ *kgm/s*.

Compare the <u>momentum lost</u> by one cart and the <u>momentum gained</u> by the other cart.

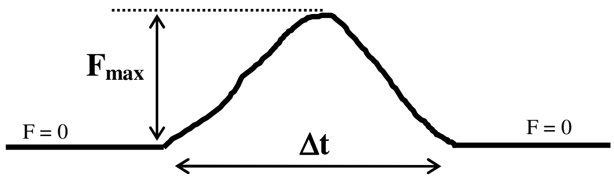
Calculate the *average force* F_{12} on cart 1 (due to cart 2) and the *average force* F_{21} on cart 2 (due to cart 1) during the time of interaction. Hint: Use Newton's Second Law. Be careful with signs. Show your calculation in the space below:

F_{12} = ____________ N. F_{21} = ____________ N.

<u>CONCLUSION</u>:

Force Curves

During the interaction, the magnetic force on each cart varies with time. This time-dependent force function starts at zero (when the carts are far apart), then rapidly rises (as the carts get closer), then reaches a maximum (when the carts almost touch), and then falls back to zero (as the carts move farther apart). The force on cart 2 as a *function of time* has the following overall shape:



Note that the force curve is characterized by two parameters (height and width): the *maximum force* F_{max} and the *interaction time* Δt. The force F_{max} occurs when the carts are closest to each other with their magnets repelling most strongly. As always, Δt is the time interval during which one cart feels the force due to the other cart.

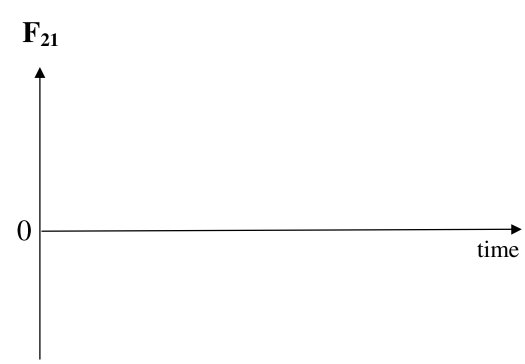
What is the exact shape of this *force curve*? Since F is equal to ma, and m is a constant, the shape of the F curve is *identical* to the shape of the a curve, i.e. $a_1 = F_{12}/m$ and $a_2 = F_{21}/m$. CHANGE the time scale on each a graph to match the time scale on the corresponding v graph. Summarize the deep (symmetrical) relationship between the overall shapes of your a_1 and a_2 curves:

Calculate the maximum forces (*action* and *reaction*) during the magnetic interaction:

$F_{12\,max}$ = $m_1\, a_{1\,max}$ = ( ) ( ) = ______________ *N.*

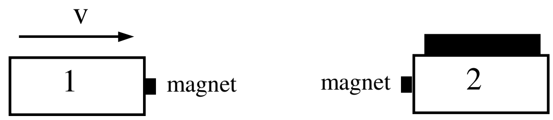
$F_{21\,max}$ = $m_2\, a_{2\,max}$ = ( ) ( ) = ______________ *N.*

Sketch your force curves below. Write the numerical values of the height and width on each curve.



Experiment 2. Magnetic Interaction (Unequal Masses)

Make cart 2 more massive than cart 1 by placing the bar weight on top of cart 2. Give cart 1 an initial velocity toward cart 2 (initially at rest). Record the motion of each cart.



PRINT your graphs. Write the numerical values of the *initial* velocity, the *final* velocity, and the *change* in velocity of each cart directly on the corresponding parts of your velocity curves.

Momentum Balance

Momentum Change of Cart 1:

$m_1 \Delta v_1$ = (*kg*) (*m/s*) = ______________ *kg m/s.*

Momentum Change of Cart 2:

$m_2 \Delta v_2$ = (*kg*) (*m/s*) = ______________ *kg m/s.*

Force Curves

Calculate the maximum forces during the magnetic interaction:

$F_{12\,max}$ = $m_1\, a_{1\,max}$ = ( ) ( ) = ______________ *N.*

$F_{21\,max}$ = $m_2\, a_{2\,max}$ = ( ) ( ) = ______________ *N.*

CHANGE the t scale on each *a* graph to match the corresponding t scale on each v graph. Your two *a* graphs should exhibit Newtonian third-law symmetry. Sketch your measured force curves below. Write the numerical values of the height and width on each curve.

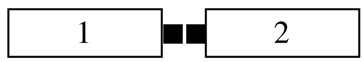
Based on your **F** graphs, what does your experiment prove?

Part III.   Conservation of Momentum

What does *"Action = Reaction"* have to do with *"Momentum = constant"* ?   Everything! From your experimental tests of *Newton's Third Law*, you discovered that the "amount of mv" lost by one object is equal to the "amount of mv" gained by the other object. This means that the "total amount of mv" in the system cannot be created or destroyed – it is conserved!

<u>Experiment 1</u>.   Create Velocity Reaction

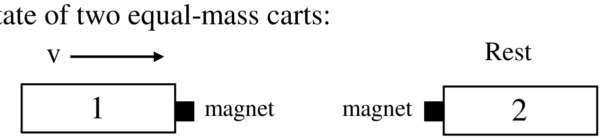
Here is the initial state of the system:



Push the equal-mass carts together with your hands until the magnets on each cart "touch". In this initial state, the magnetic repulsion is ready to "explode" the system. Release both carts at the *same time* and observe them "fly apart". Based solely on your observation (no quantitative measurements), sketch three momentum curves (*momentum as a function of time*): p_1 of cart 1 , p_2 of cart 2 , and $p_1 + p_2$ of the system. Label your three curves. Neglect friction.

<u>Experiment 2</u>.   Velocity Exchange Reaction

Here is the initial state of two equal-mass carts:



Based solely on observing this reaction, sketch and label p_1 , p_2 , and $p_1 + p_2$ as time flows.

Part IV. Design Project. Reduce Speed by 2/5

Here your team will study a one-dimensional, completely-inelastic collision between two carts. Before the collision, cart 1 moves with some initial speed toward cart 2. Cart 2 is initially at rest.

The DESIGN SPECS require:

1. **After the collision, the two carts must remain stuck together.**
2. **The final speed of the joined carts must equal 2/5 the initial speed of cart 1.**

Theory

Derive the *theoretical relationship* between the cart masses, m_1 and m_2 , that will give the 2/5 speed reduction. Work out your theory in the space below.

Equipment

How do you plan to get the carts to stick together? What are the masses (in kilograms) of the two carts you plan to use to test your theory?

Experiment

Perform the collision experiment with the two-cart system you designed. Use a sensor to record the motion. PRINT your velocity vs time graph. Change the graph scales so that the initial and final speed "plateaus" are clearly displayed . WRITE the values of the initial and final speeds on your graph. In the space below, compare your experimental data with the design specs.

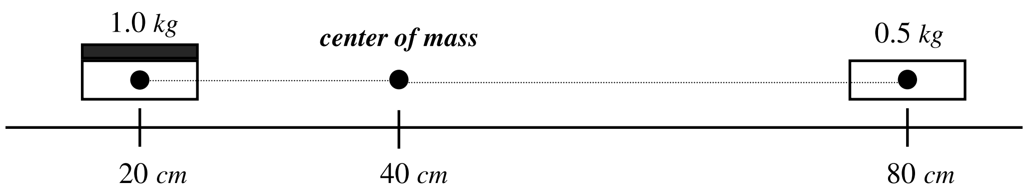
Part V. The Center of Mass: A Case Study

What is the Center of Mass?

Place cart 1 (loaded with the bar weight) on the track so that the center of cart 1 is at $x_1 = 20$ *cm*. The x axis is defined by the meter scale fixed to the track. Place cart 2 (no added weight) so that its center is at $x_2 = 80$ *cm*. The masses of the carts are $m_1 = 1.0$ *kg* and $m_2 = 0.5$ *kg*.

The <u>*center of mass*</u> x_{cm} of this system of two carts is

$$x_{cm} = \frac{1.0\,kg \bullet 20\,cm + 0.5\,kg \bullet 80\,cm}{1.0\,kg + 0.5\,kg} = 40\,cm\,.$$



Discovering a Conservation Law

Now attach a spring between the carts (from the *right* side of the 1.0 kg mass to the *right* side of the 0.5 kg mass). Make sure the Velcro patches are facing each other so that the carts stick together when they collide. Pull the carts apart so that their centers are at 20 *cm* and 80 *cm* as before. This is the *initial state*.

Release the carts at the *same time*. Observe the motion. The carts accelerate toward each other, collide, and come to rest (*final state*). Record the final resting positions (x_1 and x_2) of the *center* of cart 1 and the *center* of cart 2 in the table below. Calculate the center of mass x_{cm} of the system of two carts in this *final* state. SHOW your calculation in the space below:

	m_1	m_2	x_1	x_2	x_{cm}
initial	1.0 *kg*	0.5 *kg*	20 *cm*	80 *cm*	40 *cm*
final	1.0 *kg*	0.5 *kg*			

Does your experimental data prove that the center of mass is *conserved* ? i.e. Does x_{cm} (*initial*) equal x_{cm} (final) within experimental error (10%) ?

The Center of Mass is the Place to Be

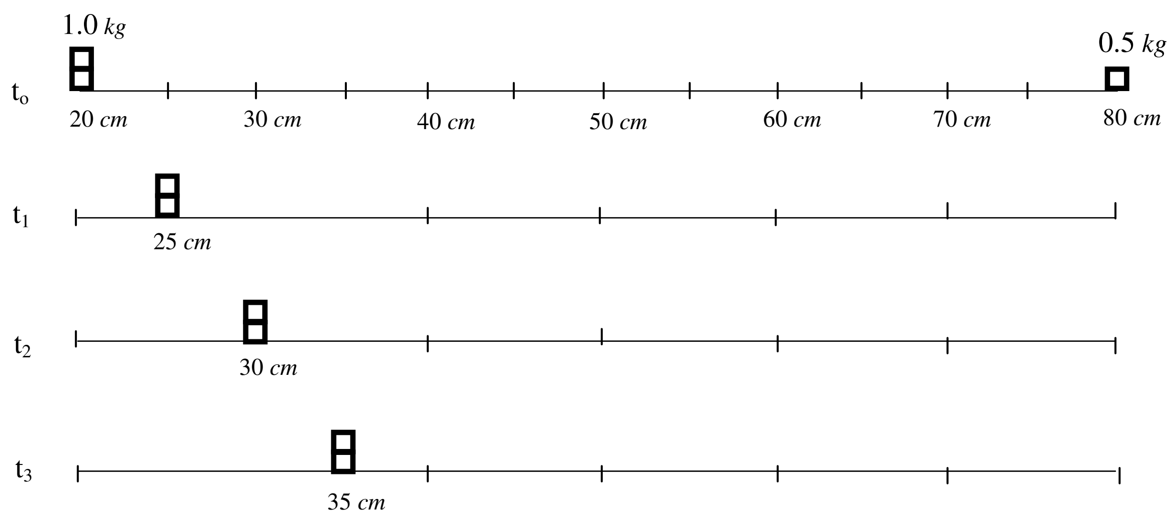
Why is x_{cm} conserved? Because $F_{12} = - F_{21}$! Here is the Newtonian logic: "$F_{12} + F_{21} = 0$" implies "$m_1 a_1 + m_2 a_2 = 0$" which says "$a_{cm} = 0$".

Amidst the "*chaos*" of accelerating parts (the carts rushing toward each other), there is one point of "*calmness*": the center of mass x_{cm} .

i.e. x_1 and x_2 are complicated (depend on time t). $m_1x_1+m_2x_2$ is simple (independent of t).

The conservation relation " x_{cm} = *constant* " allows you to know the position of each cart at any time during the motion without having to know anything about the time-dependent internal (spring) force that is causing the motion!

The initial position 20 *cm* of cart 1 (1.0 *kg*) and the initial position 80 *cm* of cart 2 (0.5 *kg*) are marked on the position diagram below. The position of cart 1 is also marked at three later times of the motion. Mark the position of cart 2 at these same three future times. Mark the position of the center of mass of the system at all four times with the symbol ×.



Einstein , Center of Mass , E = mc²

The two Conservation Laws that you discovered in this laboratory, " $m_1v_1 + m_2v_2$ = *constant* " and " $m_1x_1 + m_2x_2$ = *constant* " , are two of the most important consequences of *Newton's Third Law* " $F_{12} + F_{21} = 0$ " .

Albert Einstein realized the deep significance of the constancy of x_{cm}. In his 1906 paper, "*The Principle of Conservation of Motion of the Center of Mass and the Inertia of Energy*", Einstein used the relation x_{cm} (initial) = x_{cm} (final) to derive the most famous equation in science: $\mathbf{E = mc^2}$. Einstein used this center of mass theorem to analyze the motion of a train car when a light bulb inside the car was turned on. The light source emits "photons" to the right which causes the train to recoil to the left. The center of mass does not move!