Team: ________________

____

Rotational Motion

Rotational motion is everywhere. When you push a door, it rotates. When you pedal a bike, the wheel rotates. When you start an engine, many parts rotate. Electrons rotate in an atom. H_2O molecules rotate in a microwave (and thereby cook the food!). Galaxies rotate in the Universe.

<u>All motion</u> can be classified into three basic types: *Translation* , *Rotation* , and *Vibration*. A baseball *translates* along a parabolic path, *rotates* (spins) about its center, and *vibrates* when it hits a bat. The earth *translates* around the sun in an elliptical path, *rotates* about its axis, and *vibrates* during an earthquake. *"Translation* and *Rotation"* is the name of the game for gymnasts, high divers, and ice skaters. The *translation* and *rotation* of N_2 and O_2 molecules determines the temperature and thermodynamics of the atmosphere – and therefore the weather.

So far, you have studied *translational motion*. Here you will explore the physics of *rotational motion*.

Part I. Angular Position, Velocity, and Acceleration

In previous labs, you used a *motion sensor* to measure the *translational* (linear) variables x , v, a. Here, you will use a *rotary motion sensor* to measure the *rotational* (angular) variables θ , ω , α.

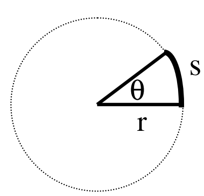
Definition of Motion Variables

Variable	*Translational Motion*	*Rotational Motion*
Position	x	θ
Velocity	$v \equiv dx/dt$	$\omega \equiv d\theta/dt$
Acceleration	$a \equiv dv/dt$	$\alpha \equiv d\omega/dt$

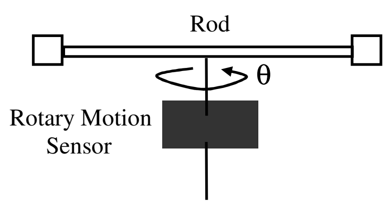
A. Angular Position θ

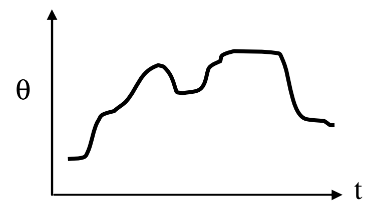
The angle θ is the natural quantity to represent the position of a solid object rotating about a fixed axis. This position variable has units of *radians* and is defined as the ratio of two *lengths* involving a circle: *arclength s* over the *radius r*. Note that θ = 1 *radian* is defined by s = r.

$$\theta \equiv s/r$$



The rotary motion sensor measures θ as a function of time t. Start Logger Pro and open the file *Torque*. Activate the sensor by clicking on *Collect* or pressing the *Space Bar*. Gently rotate the rod with your hand and watch the angular position graph recorded by the sensor. Rotate the rod clockwise and counter-clockwise, move it fast, move it slow, hold it at rest, let it spin freely and note how each of these different rotational motions are displayed on the θ(t) graph – the angular *"worldline"* of the rod.





Click *Collect* again. Starting from rest, rotate the rod with your hand. Stop the rod when the rod has made one revolution (one complete circle). Report the change in the angular position of the rod recorded by your rotary motion sensor.

$\theta_f - \theta_i$ = ____________ – ____________ = ______________ *radians*.

What is the value of this angular displacement in *degrees*? Show your calculation that converts *radians* into *degrees*.

$\theta_f - \theta_i$ = ______________ *degrees*.

B. Angular Velocity ω

Angular velocity is the rate of change of angular position with respect to time: $\omega \equiv d\theta/dt$. Graphically speaking, ω is the slope of the θ(t) curve.

Give the rod a gentle push with your hand and then let go. With no force (torque) acting on the rod, except for negligible friction, the angular speed ω will remain constant. Both *you* and the *computer* will monitor the motion. Click on *Collect*. While the rotary motion sensor is collecting θ(t) data, use your *stopwatch* to measure the time Δt it takes for the rod to complete five revolutions. Find ω two different ways:

1. *Stopwatch Measurement*.

$\Delta\theta/\Delta t$ = ____________ rad / ____________ s = ____________ *rad/s.*

2. *Motion Sensor Data*. The θ(t) graph should be a straight sloping line (since ω is constant). Select the good (straight line) data region of your θ(t) graph. Find the best-fit line through the data. Record the slope of the line.

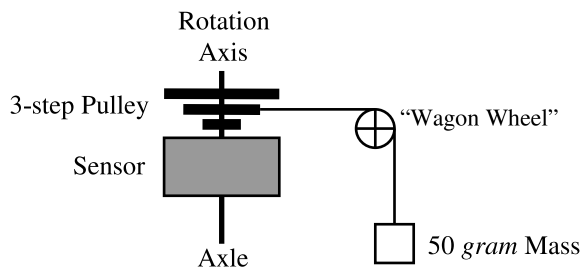
Slope of θ(t) Line = ____________ *rad/s* .

Percent Difference between "Δθ/Δt" and "Slope of θ(t)" is ____________ % .

C. Angular Acceleration α

Angular acceleration is the rate of change of angular velocity with respect to time: $\alpha \equiv d\omega/dt$. Graphically speaking, α is the slope of the $\omega(t)$ curve.

Note that the spinning axle of the rotary motion sensor coincides with the rotation axis of the system. This axle goes through the center of a pulley located directly above the sensor. This pulley has three "steps", each with a different radius. Wrap the thread around the middle step (step 2) of this three-step pulley. Feed the thread over the plastic "wagon wheel" clamped to the sensor. This wheel merely changes the *direction* of the thread (tension force) from horizontal to vertical. When we say "*pulley*" in this lab, we are referring to the 3-step pulley around the rotation axis and <u>not</u> to the plastic wheel.



CAUTION: Make sure that the two brass weights are tightly secured on the ends of the rod. Attach a 50 *gram* mass to the end of the thread. Release the mass. Record the motion. Note that your $\theta(t)$ graph is *curved* ! More precisely, $\theta(t)$ is a parabola. Remember: a *curving* position-time graph is the kinematic trademark of an *accelerating* system. Your $\omega(t)$ graph should be a straight sloping line because the acceleration of the system – which is due to the constant force of gravity – is constant. Select the good data region of your $\omega(t)$ graph. Find the best-fit line and report the slope:

α = slope of $\omega(t)$ line = ___________________ rad/s^2 .

Connection between Translation and Rotation – a Thought Experiment

With a different pulley in your experiment, you find the following result:

In **10** *seconds*, the hanging mass *falls* **10** *centimeters* from rest and the pulley *rotates* **10** *radians*. Calculate the radius and the angular acceleration of the pulley.

R = ____________ .

α = ____________ .

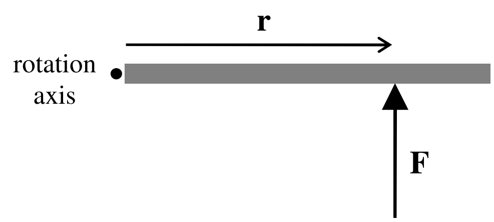
Part II. Torque and Angular Acceleration

Force causes *linear* acceleration: $F = ma$. *Torque* causes *angular* acceleration: $\tau = I\alpha$.

A. Basic Principles

The ability of a force to rotate an object about an axis depends on two variables:

1. The magnitude of the *force F*.
2. The *distance r* between the axis of rotation and the point where the force is applied.



Try opening a door by applying the *same* force F at *different* distances r from the hinge. You will quickly realize that the resulting motion of the door – the acceleration α – depends on *F and r*. It turns out that the "*turning ability*" of a force is simply the product of F and r. The technical name for this turning ability is *torque*.

Definition. The *torque* **τ** exerted by a force **F** that is applied at a point **r** relative to the origin is the cross product of **r** and **F**: $\boldsymbol{\tau} \equiv \mathbf{r} \times \mathbf{F}$.

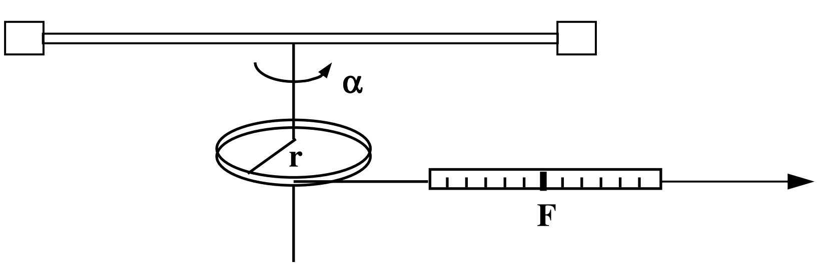
The deep law of physics is that torque is directly proportional to angular acceleration: $\tau \sim \alpha$. τ does not go like α^2 ! The proportionality constant between τ and α is the rotational inertia I, defined as $I \equiv \sum mr^2$, i.e. the sum of mr^2 over all the mass elements (particles) in the system.

Theorem: The fundamental equation of rotational motion is $\tau = I\alpha$.

Proof: Multiply $F = ma$ by r to get $rF = mra$. Use $a = r\alpha$ to get $rF = mr^2\alpha$. Sum over particles to get $\Sigma rF = (\Sigma mr^2)\alpha$, which is $\tau = I\alpha$.

B. Apply the Torque τ , Measure the Acceleration α , Prove the Law τ = Iα

You will apply a *torque* τ to the pulley by pulling on the thread. This torque will cause the pulley – and everything attached to the pulley (axle, rod, brass weights) – to undergo rotational motion with *angular acceleration* α. You will measure τ with the spring scale (F) and a ruler (r). You will measure α with the rotary motion sensor. You will discover firsthand the relation between τ and α.



Apply τ . Measure α .

Make sure the two brass weights are tightly secured on the ends of the rod – as far away as possible from the axis of rotation.

Make sure that the spring scale reads 'zero' when it is in the horizontal position and nothing is attached to the small hook. Remove the hanging mass from the thread and attach the small hook of the spring scale to the thread. Pull the scale so that the scale reads a *constant force* F during your entire pull. Make sure to keep the thread and scale horizontal while you are pulling. Lower the long vertical rod on which the rotary apparatus is mounted to a level that makes it easier to read the spring scale and achieve a horizontal pull. Maintaining a constant force may be a bit tricky. It is okay if the force fluctuates slightly – sometimes you pull too hard, sometimes too soft, but the average will be just right! Note that the scale reading gives the *tension* in the thread. The torque due to the tension force F acting at the distance r from the rotation axis is rF.

While you are pulling, record the motion using the rotary motion sensor. Find the angular acceleration α of the system by finding the slope of the best-fit line through the velocity $\omega(t)$ data. Be sure to select the *good data region* of your $\omega(t)$ graph – the region where $\omega(t)$ is a straight *line*, i.e. where the slope is constant.

Report your results in the *τ and α Table*. Do this measurement for three values of the force (F = 0.1N , 0.3N , 0.5N) on the middle-step pulley and three values of the force (F = 0.1N , 0.3N , 0.5N) on the largest-step pulley. Measure the radius r of each step using a short ruler.

Hints for measuring r: Do not remove the 3-step pulley from your apparatus. You will find an identical pulley on the back table to take measurements. Measure the diameter. Try to compensate for the fact that the wound thread makes the radius slightly larger.

τ and α Table

r (*m*)	F (*N*)	τ = rF (*Nm*)	α (*rad/s*^{2})
	0.1		
	0.3		
	0.5		
	0.1		
	0.3		
	0.5		

Analysis of τ-α Data

1. Look at the first two rows in your *τ and α Table*. Note how the force (and torque) increases by a factor of three (triples) in going from 0.1 *N* to 0.3 *N*. By what factor does the corresponding angular acceleration increase? Compare the factors by which the torque and angular acceleration increase in going from the fourth row to the sixth row.

2. In order to test the Newtonian relation $\tau \sim \alpha$, you could plot τ versus α. If Newton is right, then what is the "shape" of the resulting $\tau(\alpha)$ graph that you would expect?

3. Use *Graphical Analysis* to plot τ (y axis) versus α (x axis). Include the point (0, 0) on your graph. Simply by looking at the overall "shape" defined by the seven data points (α, τ) can you claim to have experimentally "proved" the fundamental theoretical relation of rotational dynamics: " τ is directly proportional to α " ? Explain. Can you rule out the relation $\tau \sim \alpha^2$?

4. There should be two points on your graph that are very close together – essentially "on top of each other". These two points correspond to the *same angular acceleration* (approximately), but to completely *different applied forces*. Write the values of α and F at these two points:

F_1 = __________ *N* . α_1 = ___________ *rad/s*^{2} .

F_2 = __________ *N* . α_2 = ___________ *rad/s*^{2} .

Explain how these different *forces* ($F_1 \neq F_2$) can cause the same *motion* ($\alpha_1 \approx \alpha_2$) .

5. The physics equation $\tau = I\alpha$, written as "$\tau = I\alpha + 0$", has the same mathematical form as the generic equation of a line "y = mx + b". If your $\tau(\alpha)$ graph does not look linear, then consult with your instructor. Find the best-fit line through your seven data points (α, τ). PRINT your graph showing the best-fit line along with the values of the slope and the intercept. Report the equation of your line here:

τ = ________________ α + _______________ .

What is the *rotational inertia* of your system (rod + brass weights + axle)?

I = ________________ *kg m*^{2} .

Part III. Rotational Inertia

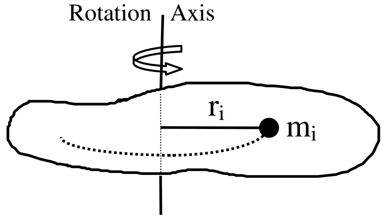
Rotational inertia I has two meanings: $I = \tau/\alpha$ and $I = \sum mr^2$. In Part II, you focused on the *dynamic* meaning of I in terms of *force* and *motion* (τ/α). Here you will investigate the *geometric* meaning of I in terms of *mass* and *distance* (mr^2).

The "Mass Distribution" Meaning of I

Translational Inertia is $M = m_1 + m_2 + m_3 + \ldots$

Rotational Inertia is $I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots$

In the I-equation, r_i is the *perpendicular distance* of the mass element m_i from the *axis of rotation*. Note that the rotational inertia I of a rotating body depends on *how the mass is distributed about the axis of rotation*. An object whose mass is located far away from the axis of rotation has a larger I − and thus is harder to turn (accelerate) − than an object with the same mass that is distributed close to the axis of rotation.



Together, the "amount of mass" $M = \sum m$ and the "distribution of mass" $I = \sum mr^2$ tell you everything about the linear and angular response of a system to external forces and torques:

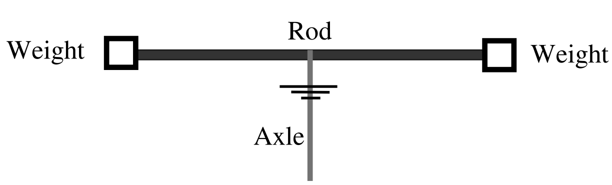
M measures an object's resistance to changing its *linear* velocity: $F\Delta t = M\Delta v$.
I measures an object's resistance to changing its *angular* velocity: $\tau\Delta t = I\Delta\omega$.

Where is the "<u>I Scale</u>" ?

When an object is placed on the mass scale, or "M-scale", the scale reads the *translational* inertia ($M = \sum m$) of the object. Unfortunately, there does not exist an "*I-scale*" that directly reads the *rotational* inertia ($I = \sum mr^2$)! You will have to find I by using the M-scale to measure the *amount* of mass and a ruler to measure the *shape* of the mass, i.e. the relevant *distances* from the rotation axis.

The Experiment: Measure "mr^2 of the Parts" to find "I of the Whole"

Your goal is to find the value of $I \equiv \sum mr^2$ that characterizes the rotational inertia of your system. *Everything* (each mass element) that is rotating in the system contributes to the rotational inertia. As the following schematic shows, your system consists of four rotating parts:



Since I is the sum over all the moving "mr^2" in your system, the total value of I is equal to the sum over the I values of the separate moving parts:

$$(\Sigma mr^2)_{System} = (\Sigma mr^2)_{Weights} + (\Sigma mr^2)_{Rod} + (\Sigma mr^2)_{Axle}$$

$$I = I_{Weights} + I_{Rod} + I_{Axle}$$

$$= 2MD^2 + \frac{1}{12} M_R L^2 + \frac{1}{2} M_A R^2 .$$

Measure the relevant *mass* and *distance* for each of the three parts: M and D , M_R and L , M_A and R . Do not remove these parts from your apparatus. Identical parts are located on the table in the back of the room where the mass scale is located.

Mass of one brass weight: M = ____________ *kg*.

Distance between axis of rotation and *center of mass* of brass weight: D = ____________ *m*.

Mass of Rod: M_R = ____________ *kg*. Length of Rod: L = ____________ *m*.

Mass of Axle: M_A = ____________ *kg*. Radius of Axle: R = ____________ *m*.

Compute the total I of your system in the space below:

Conclusion

Write your measured values of I that you obtained using two different methods (Parts II and III):

Torque / Acceleration Method ($I = \tau/\alpha$) : I = ____________________ $kg\ m^2$.

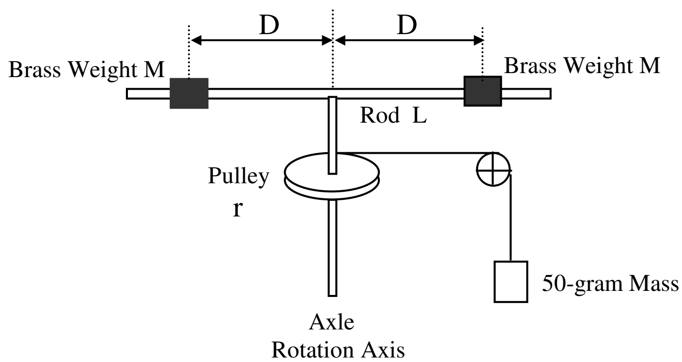
Mass • Distance^2 Method ($I = \Sigma mr^2$) : I = ____________________ $kg\ m^2$.

Compare your two values of I. Which method of measuring I do you think is more accurate? Explain.

Part IV. Design Problem – Tuning I so that "70 in 7.0 ± 0.7"

Your goal is to use your *rotating machine* to lower a 50-*gram* mass, starting from rest, through a distance of 70 *centimeters* in a time of 7.0 ± 0.7 *seconds*. Use the second-step pulley. The only machine parameter that you can adjust is the location of the brass weights along the rod.

Here is a schematic of the rotating machine:



The Theory

The following set of questions guide you through the theoretical analysis to find the value of D ≡ *distance between the rotation axis and the center of mass of each brass weight*. Show all your work in answering each question.

1. Calculate the acceleration a of the hanging mass.

a = _____________ m/s^2 .

2. Calculate the tension T in the thread. First draw a free body diagram for the hanging mass and then set up F = ma for the translational motion of the mass.

T = _____________ N .

3. Calculate the torque τ on the pulley due to the tension T.

τ = _____________ *Nm* .

4. Calculate the angular acceleration α of the rotating system.

α = _____________ *rad/s*^{2}.

5. Calculate the rotational inertia I of the system.

I = _____________ *kg m*^{2} .

6. Calculate the contribution to I from the two brass weights.
   Hint: $I = I_{Weights} + I_{Rod} + I_{Axle}$.

$I_{Weights}$ = _____________ *kg m*^{2} .

7. Calculate D.

D = _____________ *m* .

The Experiment

Place the brass weights on the rod at the location predicted by your theoretical value of D. CAUTION: make sure each brass weight is *fastened tightly* to the rod. Start the 50-*gram* mass at a height of 0.70 *meter* above the table. Release the mass from rest. Use a *stopwatch* to time how long it takes for the mass to fall and hit the table. Repeat three times. List your three values of time. Report the average value.

t = ______________ *seconds* .

Does your measured value of t fall within the 10% window (7.0 ± 0.7 *seconds*) allowed by the *Design Specs* ? If not, consult with another team or your instructor.