Team ______________

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Oscillations

Oscillatory motion is motion that repeats itself. Oscillations are everywhere: guitar string, pendulum, piston, neutron star (pulsar), drumhead, nuclear magnetic resonance (NMR), alternating current, neutrino oscillations, electron cloud in an atom, vibration of atoms in a solid (heat), vibration of molecules in air (sound), vibration of electric and magnetic fields in space (light), and the oscillating universe (big bang-big crunch).

The Force

The dynamical trademark of all oscillatory motion is that the net force causing the motion is a *restoring force*. If the oscillator is displaced away from equilibrium in any direction, then the net force acts so as to restore the system back to equilibrium.

Definition: A *simple harmonic oscillator* is an oscillating system whose restoring force is a *linear force* – a force F that is proportional to the displacement x :

$$F = -kx \ .$$

The *force constant* k determines the strength of the force and measures the "springiness" or "elasticity" of the system. Slinkys, long pendula, and loose drumheads have small k values. Car coils, short pendula, and tight drumheads have large k values.

The Motion

The motion of *all* mechanical systems is determined by Newton's law of motion $F = ma$. For any simple harmonic oscillator system, characterized by *mass* m and *force constant* k, the equation of motion is $-kx = ma$, or

$$a = -(k/m)\,x \ .$$

The solution of this equation of motion is

$$x(t) = A \sin\left(\sqrt{\frac{k}{m}}\; t\right) \ .$$

To see that this is the solution, note that the second derivative of x(t) with respect to time, which is the acceleration $a(t)$, is equal to $-(k/m)\,x(t)$. The ratio of the *inertia* (m) of the system to the *elasticity* (k) of the system has units of $[seconds]^2$ and sets the scale of time for all simple harmonic motion. It follows from this sinusoidal x(t) function that the **period** of the motion – the time required to complete one oscillation – is $T = 2\pi\sqrt{m/k}$.

Summary: The Essence of Simple Harmonic Motion

IF the force acting on mass m has the *linear* (simple) form: $F = -kx$

THEN the motion of the mass will be *sinusoidal* (harmonic): $x = A\sin(2\pi t/T)$

AND the period T depends solely on the *mass/force* ratio: $T = 2\pi\sqrt{m/k}$

Part I. The Pendulum Oscillator

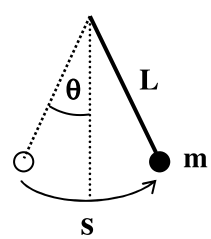
A simple pendulum consists of a mass m suspended from a lightweight cord of length L in a gravitational field g,

A. Galileo's Discovery

Galileo made the first quantitative observations of pendulum motion in 1583 by observing the swing of a chandelier hanging from the cathedral ceiling in Pisa, Italy. He used his pulse to measure the period. His observations and theory led to the construction of the first pendulum clock by Christiaan Huygens in 1650.

The Galilean Question: How does the period T of a pendulum – the time required to complete one back and forth swing – depend on

1. The size **s** of the arc?
2. The mass **m** of the weight?
3. The length **L** of the cord ?



For example, if a pendulum takes 1 sec to swing through a 10 *cm* arc, then shouldn't the same pendulum take 2 sec to swing through a 20 *cm* arc? In Experiments 1, 2, and 3, you will discover the answers to Galileo's questions.

*****Apparatus Note*: To set the length of the pendulum, simply slide the string between the metal brackets until you achieve the desired length and then tighten the screw. No need to cut string.

Experiment 1. Quadruple the Size of the Arc

Construct a pendulum with m = 200 *grams* and L = 1.0 *meter*. Remember to measure L from the pivot point to the center of mass of the weight. Pull the pendulum sideways so the cord makes an angle θ with the vertical and release. Use a stopwatch to measure the period T for $\theta = 5^o$ and 20^o. For accuracy, measure the time for five back-and-forth swings and then divide by five to get the period.

Results:

Conclusion:

Experiment 2. Quadruple the Mass of the Weight

Keep L = 1.0 *meter*. Measure T for m = 100 *grams* and 400 *grams*.

Results:

Conclusion:

Experiment 3. Quadruple the Length of the Cord.

Keep m = 200 gram. Measure T for L = 1/4 *meter* and 1 *meter*.

Results:

Which formula best fits your experimental result on the relation between period and length:

$T \sim L$ $\quad$ $T \sim 1/L$ $\quad$ $T \sim L^{1/2}$ $\quad$ $T \sim L^{3/4}$ $\quad$ $T \sim L^2$

Justify your formula selection:

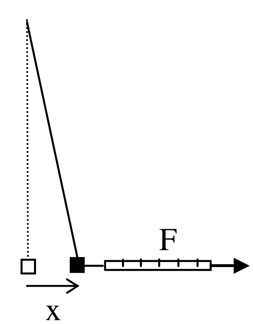
B. The "Springiness" of a Pendulum

Finding the force constant of an oscillatory system is an important quest for scientists and engineers. If you know the k value of a system, then you know almost everything about the system!

Measure k. Here you will measure the *force constant* k of a pendulum. Construct a pendulum of mass m = 200 *grams* and length L = 1.0 *meter*. Lay the meter stick on the table. The mass should hang directly above and almost touch the stick. Zero the spring scale while it is horizontal. Attach the spring scale to the bottom of the hanging mass. Use the spring scale to displace the mass a distance x = 5 *cm* away from its equilibrium position. While you keep the mass at rest in its displaced position at 5 *cm*, read the value of the force F on the scale. Since the mass is at rest, the restoring force of gravity pulling the mass back to equilibrium is equal and opposite to the pulling force of the scale. Repeat for x = 10 *cm* and x = 15 *cm*.

For each displacement x, record the corresponding force F:

x (*m*)	0.05	0.10	0.15
F (*N*)			



According to the theoretical definition of "simple harmonic oscillator" on page 1, is your pendulum a *simple harmonic oscillator* within experimental error? Carefully explain your answer.

What is the force constant k that characterizes the "springiness" of your pendulum?

k = _____________ *N/m.*

What is the m-over-k parameter of your pendulum? m/k = _____________ .
(Don't forget to state the units)

Compute T. Use your measured m/k parameter to calculate the period T of your pendulum according to the theoretical "m/k formula" derived in the introduction.

T_{theory} = _____________ *seconds.*

In part A, you directly measured the period of a pendulum (m = 0.20 *kg*, L = 1.0 *m*) using a *stopwatch*. Compare this measured value with your calculated value T_{theory}.

C. The Famous "L over g" Formula" for a Pendulum

The formula that reveals how the period T of a simple pendulum depends on its length L (for small arcs) is

$$T = 2\pi\sqrt{\frac{L}{g}} \ .$$

This pendulum formula is a special case of the general formula: $T = 2\pi\sqrt{\frac{m}{k}}$.
This "m/k formula" is valid for ALL harmonic oscillators.

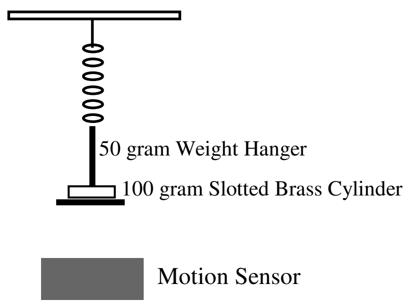
In order to derive the "L/g formula" from the "m/k formula", what must be the force constant k of a pendulum in terms of m , L , and g ?

k = ______________ .

Show that this theoretical expression for k is consistent with your measured value of k in part B.

Part II. The Mass-Spring Oscillator

Suspend the 50-*gram* weight hanger from the end of the narrow silver spring. Place the 100-*gram* slotted brass cylinder on the hanger platform. Place the motion sensor on the table directly under the oscillator. The distance between the sensor and the resting oscillator should be about 25 *cm*. Open Logger Pro file *DisVelAcc*. With the mass in the resting (equilibrium) position, *zero the motion sensor* by clicking *Experiment* on the menu bar and selecting *Zero*. Pull the mass about 10 *cm* away from the equilibrium position (x = 0) and release. Record the oscillations with the motion sensor. If needed, change the scales on your graph so that you can clearly see the sinusoidal variations.



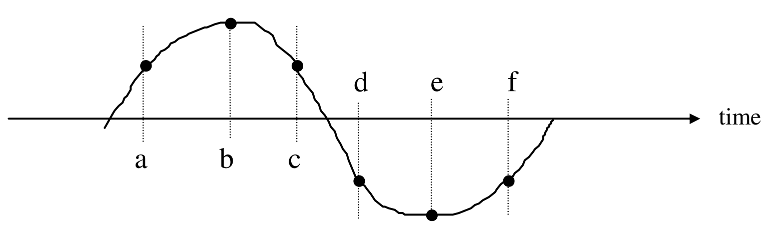
A. x(t) and *a*(t) are "In Sync". Finding T from x(t) / *a*(t).

According to the motion equation on page 1, the kinematic trademark of simple harmonic motion is that the position x(t) and the acceleration *a*(t) are proportional to each other, and opposite in direction, for *all time* t:

$$a(t) = -\omega^2 x(t) .$$

The proportionality constant between *a*(t) and x(t) is the square of the angular frequency $\omega = 2\pi/T$ of the system. When x is zero, *a* is zero. When x is maximum (minimum), *a* is minimum (maximum). Note that the velocity is completely out of sync (out of phase) with x and *a*. When x is zero, v is max.

Examine your graphs to be a first-hand witness to this *"in-sync dance"* between x and *a*. Do your x(t) and *a*(t) curves satisfy the motion equation $a(t) = -\omega^2 x(t)$? Change the time scale to go from 0 to 3 *s*. Use the *Examine Icon* [x = ?] to find the values of x(t) and *a*(t) at the six times t = a, b, c, d, e, f shown below. Any six points that are *approximately* equally-spaced within one period of the sine curve are perfectly fine.



Record your measured values of x and *a* along with the ratio x/*a* in the following table:

t	x(t) (*m*)	*a*(t) (m/s^2)	x(t) / *a*(t) (s^2)
a			
b			
c			
d			
e			
f			

According to the theory, x(t) and *a*(t) both *vary* in time, but the ratio x(t)/*a*(t) is *independent* of t ! Your experimental data should support this theoretical fact. Based on your data, what is the constant value of x(t)/*a*(t) that characterizes your mass-spring system?

x(t) / *a*(t) = ________________ ± ______________ s^2 .

Calculate the period T of your oscillator from your measured value of x(t)/*a*(t). Remember, $a(t) = -\omega^2 x(t)$ and $\omega = 2\pi/T$. Show your calculation.

T = ______________ *s* .

B. Finding T from m/k

The mass of your mass-spring system is m = 0.150 kg Measure the force constant k of the system: hang a weight from the spring and measure the stretch. Report your k value. Find the *m-over-k* value of your oscillator.

k = ______________ *N/m* . m/k = ______________ s^2 .

Calculate the period of your oscillator from your measured value of m/k. Show your calculation.

T = ______________ *s* .

C. Summary

You have measured the period of a mass-spring oscillator using two different methods. As a third method, measure the period directly with a stopwatch. Summarize your results by listing all three of your T-values below.

Motion Method. Measure x(t) and *a*(t), compute x(t)/*a*(t) : T = ______________ *s* .

Force Method. Measure m and k, compute m/k : T = ______________ *s* .

Timing Method. Measure time directly with clock : T = ______________ *s* .

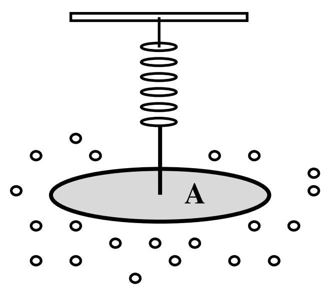
Part III. Damped Oscillator

In this experiment, you will study the effect of air friction on the motion of your mass-spring oscillator. In physics, an oscillator subject to friction is called a "damped oscillator". Friction causes the motion to decay. The amplitude of the oscillations decreases as time goes on.

Any object moving through the air experiences a 'viscous drag force' due to the myriad of impacts between the object and the air molecules. As a result of the air-object collisions, the object slows down and the air molecules speed up (the air gets hotter). *Mechanical energy* of the object is transformed into *thermal energy* of the air.

The System

Attach a cardboard surface of *area* A to your oscillator as follows: Unscrew the bottom disk from the weight hanger rod. Put the rod through the hole in the board. Screw the disk back onto the rod. Place the 100 *gram* slotted brass cylinder on top of the board. You now have a damped harmonic oscillator. Scientists and engineers use similar models to study the aerodynamic design of cars, planes and rockets, and the hydrodynamic design of ships.



As your oscillator moves up and down, imagine the cardboard surface pushing through the *"sea of air"*. The numerous collisions – about 10^{22} per second – between the surface and the air molecules (tiny "balls" of nitrogen and oxygen) are the mechanical source of the air friction.

The Experiment

Place the motion sensor directly under your oscillator. The distance between the sensor and the resting oscillator should be about 25 *cm*. Open Logger Pro file *Damped Oscillator*. With the oscillator at rest, <u>*zero the motion sensor*</u> [click on *Experiment* and select *Zero*]. Pull the mass away from the equilibrium position (x = 0) about 10 *cm* and release. Record the motion x(t).

1. Find the **half-life** $T_{1/2}$ your oscillator. Show your determination of $T_{1/2}$ directly on the printed graph of x vs t.

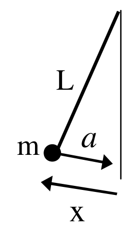
"Half-life" is defined to be the time it takes for the amplitude to decay to one-half of its initial value. The characteristic time $T_{1/2}$ is an important property of all decay processes in nature, such as oscillations, radioactivity, population statistics, drug metabolism, and chemical reactions.

2. Estimate the value of $T_{1/2}$ if you doubled the area of your cardboard surface. Explain.

Part IV. Design Project

Your goal is to design the new "Mega Pendulum Ride" at Cedar Point so that the riders feel a huge "stomach-turning" gee-force during the motion. As a prototype, you first design a small-scale model: a pendulum of length L and mass m = 200 *grams*.

The design specs require that the mass experience an acceleration $a = 1.5\ m/s^2$ when it is at a distance x = – 0.10 *m* from equilibrium.



Theory

1. Calculate the net force F on the mass when it is at x = 0.10 *m*.

F = ____________ .

2. Calculate the force constant k of the pendulum system.

k = ____________ .

3. Calculate the length L of the pendulum.

L = ____________ .

Experiment

Build the pendulum. Measure the acceleration *a* at the point x = 0.10 *m* two different ways:

Force Method. Use the spring scale to pull the mass 0.10 *m* away from equilibrium. Measure F.

Given your measured F = ___________ , compute *a* at x = 0.10 *m* in the space below.

a = ____________ .

Motion Method. Use the stopwatch to measure the period T of your pendulum.

Given your measured T = ___________ , compute *a* at x = 0.10 *m* in the space below.
Hint: $a = -\omega^2 x$.

a = ____________ .

Conclusion. How well does your pendulum prototype satisfies the theoretical design specs?

Part V. Coupled Oscillators

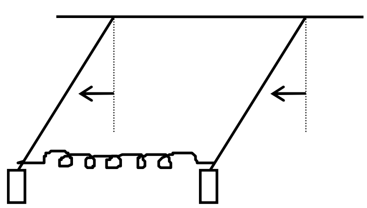
The motion of two coupled harmonic oscillators provides deep mechanical insight into all kinds of natural phenomena including:

- Quantum jump between two energy levels.
- Chemical bond between two atoms.
- Musical beats between two notes.
- *Schrödinger's Cat.*
- Vibrational modes of bridges, airplane parts, etc.
- Electric current in coupled LC circuits.

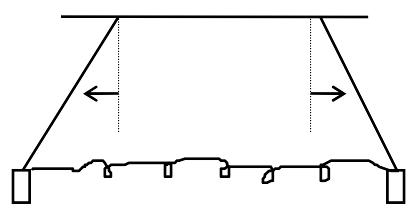
Here you will study the most famous coupled-oscillator system: two pendula coupled by a spring. You will learn how a *complicated motion* can be understood in terms of a superposition of *simple sinusoidal parts*. The simple parts are called "normal modes".

Normal Modes A and B. Examine the coupled pendula system on display in the lab.
To "excite" mode A, pull both masses away from the vertical (equilibrium) in the *same direction* and release at the *same time*. To "excite" mode B, pull both masses away from the vertical (equilibrium) in *opposite directions* and release at the *same time*. Note that in each normal mode, both masses oscillate with the <u>same</u> frequency. Measure the period T and compute the frequency of each mode. *For accuracy, use two stopwatches and measure the time for five periods.*

Mode A



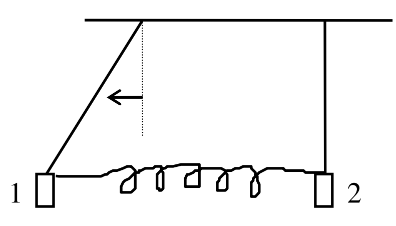
Mode B



T_A = _______________ *s* . T_B = _______________ *s* .

f_A = _______________ *Hz* . f_B = _______________ *Hz* .

Energy Exchange Motion = Mode A + Mode B. Pull mass 1 away from the vertical (do not touch mass 2) and release.

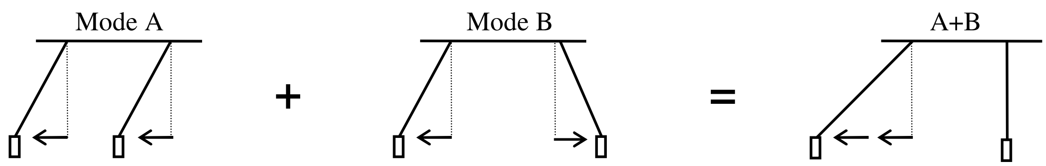


Observe this "energy exchange motion". Note that the total amount of energy in the system remains constant, but it is constantly being passed between the two pendula. Pendulum 2 speeds up at the expense of pendulum 1 slowing down, and vice versa. The energy "sloshes" back and forth between the two parts of the system. The spring provides the transfer mechanism. If there were no friction, then this exchange would go on forever! This simple system models the complex energy transfers that occur in nature.

Measure the period of the energy exchange, i.e. the time it takes for 2, which starts with zero energy, to gain all the energy from 1, and then give it all back to 1. In other words, find the time it takes for pendulum 2 to complete the cycle *rest* $\rightarrow$ *moving* $\rightarrow$ *rest*. Since the energy exchange motion is a combination of the A and B modes, we will label this motion "AB". Report your measured values of the period T_{AB} and frequency f_{AB} of this energy exchange (AB) motion. *For accuracy, use two stop watches and measure the time for three periods*.

T_{AB} = _______________ *s* . f_{AB} = _______________ *Hz* .

Relation Between f_A , f_B , f_{AB} . Why is the energy exchange motion a superposition of mode A and mode B? For a qualitative answer, the following picture illustrates how the A and B displacements add to give the energy-exchange displacement:



According to the theory (normal mode analysis), the frequency of the energy-exchange motion (AB) is related in a simple way to the frequency of the constituent modes (A and B):

$$f_{AB} = f_B - f_A .$$

How close (percent difference) do your experimental values of f_{AB} , f_B , and f_A obey this theoretical relation?

Music , Atoms , & Photons. There exists many systems and processes in nature where three frequencies are related via the "normal mode rule" $f_{AB} = f_B - f_A$. Most notably are *"beats"* in music and *"photons"* in quantum mechanics.

When two musical notes (sound waves) of slightly different frequency f_A and f_B are played together (superimposed), the result is a sound wave that pulsates at the frequency $f_{AB} = f_B - f_A$. When an electron in an atom makes a *quantum jump* from a high-energy state B, where it "orbits" at frequency f_B , to a low-energy state A where it "orbits" at frequency f_A , the atom emits a light wave of frequency $f_{AB} = f_B - f_A$. During the quantum jump, the electron is in a superposition of quantum states (modes): state A + state B.

In music, the superposition of sound waves results in *"beats"*. In quantum mechanics, the superposition of quantum waves results in a *"photon"*. The coupled pendulum system that you studied in this lab provides insight into how two tuning forks make sound (beats) and how two atomic states make light (photons).